MEASUREMENTS OF THE MULTIPLE SCATTERING OF VERY RELATIVISTIC ELECTRONS

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ABSTRACT

In order to determine the accuracy with which electron energy can be measured using multiple scattering techniques, and to compare the difference-product and cell-overlap methods of data reduction, we have multiple scattered electron tracks of known energy. A total of 163 cm of tracks of known energies of 300, 500, and 875 Mev was scattered. We found that energies calculated from our data were much lower than expected, that radiative effects could not be separated from the general depression of the energy, and that there is little difference in the two methods of data reduction _if_ certain assumptions are satisfied. We found that energy calculations using both methods compared favorably with the known energy when the noise was cell independent, and compared poorly when noise was cell dependent. However, we found that the assumption of cell independent noise was not usually valid for our data from relativistic electrons. Cell dependent noise was evident in 70% of the 300 Mev events, 63% of the 500 Mev events, and 95% of the 875 Mev events.

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1. Introduction

Nuclear research emulsions have been used as a tool in nuclear research, particularly in particle physics, for many years. Techniques for determining a particle's charge, mass, velocity, and interaction behavior are well documented by Barkas[1]. Of particular concern in recent years has been the determination of energy loss by ionization as a function of the particle's velocity in the very relativistic region. A discussion of this relationship appears in a review article published by Jongejans in 1960[2]. At that time the shape of the ionization curve was believed to look like Figure 1, where gamma = $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and the relative grain density is a measure of ionization.

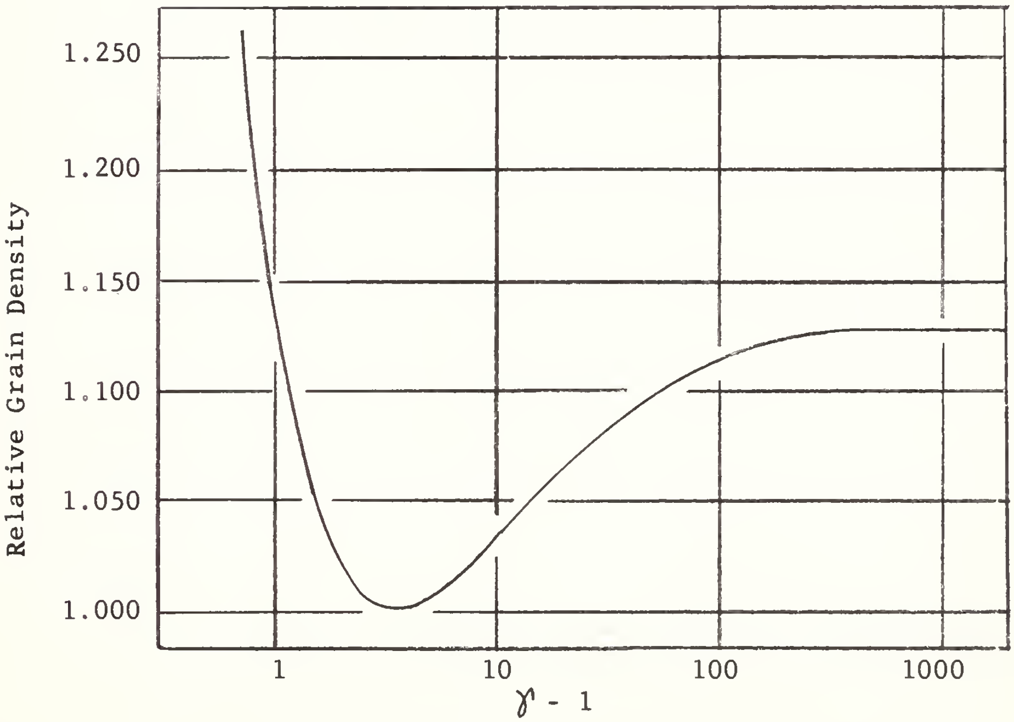


Fig. 1. Ionization Curve

In 1962 Alekseyeva et al[3] reported a drop of several percent in grain density for values of gamma greater than approximately 150. At the same time, Stiller[4] reported data which showed a slight tendency for the grain density to peak at gamma approximately 750. However, in 1964 Dyer et al[5] reported finding no significant evidence of a departure from the ionization curve described by Jongejans.

It occured to us that the conflict of data in the very relativistic region of the curve might be attributable to errors made in the pv determination of the particles. This would be especially important if particles with energies corresponding to gamma less than 100 were erroneously reported as energies corresponding to gamma greater than 100 because lower ionization values would also have been reported. We, therefore, planned an experiment to calibrate high energy points on the ionization curve, and to investigate the accuracy of multiple scattering measurements using relativistic electrons of known energy. To do this we exposed a stack of nuclear research emulsions to linear accelerator beams of 300, 500, and 875 Mev electrons.

We find that energies[1] determined from multiple scattering measurements, using both cell overlap and difference product methods, are consistently much lower than the known particle energies.

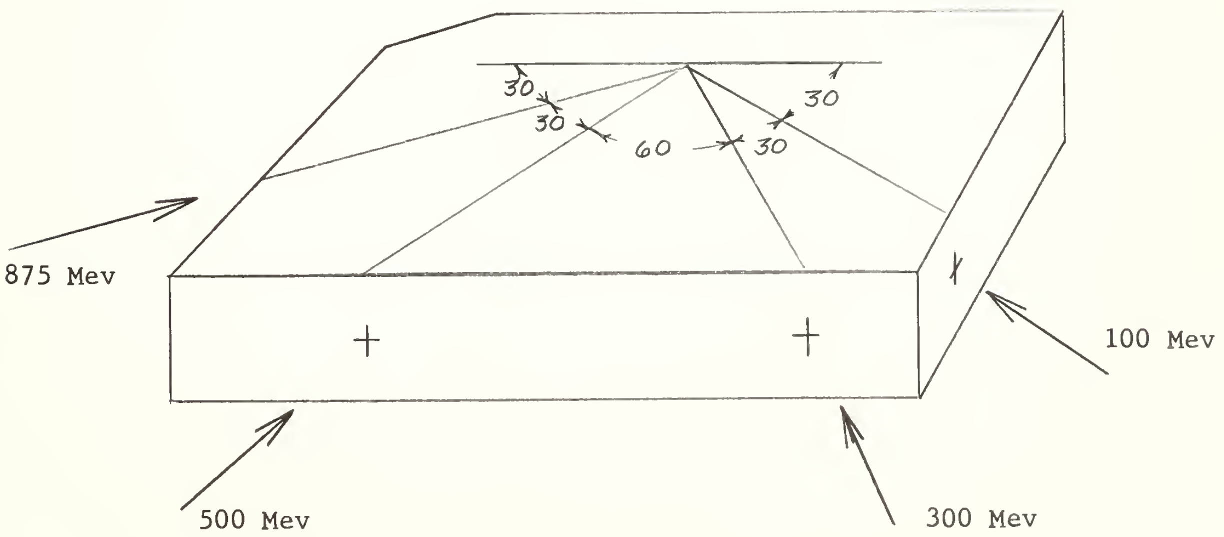
[1] For relativistic electrons, $pv \approx pc \approx E$

2. Exposure of the stacks

Two identical stacks of 8 pellicles each of Ilford K-5 emulsion were prepared. The pellicles were retangular, 3 inches by 6 inches, with one corner notched for orientation.

Beam energies of 100, 300, 500, and 875 Mev were desired to provide tracks with gamma ranging from approximately 200 to 1750. Tracks of each energy in a single pellicle would minimize effects of normalization, development, and emulsion variations inherent in drawing portions of the data from different pellicles. Therefore, each stack was exposed to beams of each of the 4 energies aimed at a point bisecting a line drawn parallel to, and 1/2 inch in, from the rear of the stack. This line is used for angular reference. See Figure 2. Then the stack was turned so that the 100 Mev beam went through at 30° to the reference line, the 300 Mev beam at 60°, the 500 Mev beam at 120° and the 875 Mev beam at 150°. Thus the tracks at different energies are all contained in a single pellicle and are easily identifiable by their entry angle. This procedure rendered the portion of the stack near the aim point useless because of the high track density as the tracks of the different energies converged, but this is acceptable because well over one radiation length (2.97 cm for emulsion) of track for each energy lies between the entry points and this saturated area.

Fig. 2. Stack Exposure Geometry



The track density was planned to be approximately 10^4 cm^{-2} for each beam energy, and a density estimated to be approximately 10^5 cm^{-2} was achieved in one stack. Unfortunately the other stack was hit with the 875 Mev beam at an intensity of approximately 10^{10} electrons cm^{-2} and was completely blackened by secondary radiation. The path of each beam in the usable stack is easily identifiable except for the 100 Mev beam which apparently missed the stack.

The exposure was made at Stanford, October 30, 1964, and the stacks were taken to UCLRL on October 31, packed in dry ice and stored until development was begun November 7, 1964. After the development, the plates were brought to USNPGS for analysis.

The plates were compared and one plate was selected for this experiment. The selection was somewhat arbitrary as only one plate from the usable stack was rejected for surface defects. All of the plates show a large random grain background and rather poor grain density for the electron tracks. This made track following difficult and probably increased the errors made in our work.

3. Scattering techniques

A charged particle passing through material undergoes many small changes in its direction as a result of coulomb forces as it passes near atomic nuclei. The technique known as "multiple scattering" is the measurement of the sum of these small deviations over a certain distance, or cell length.

The quantity pv is related to the RMS angular deviation, α, and the cell length, s, as

$$pv = \frac{K s^{\frac{3}{2}}}{\alpha},$$

where K is an appropriate constant whose value depends on the technique used to estimate α.

Angular and co-ordinate methods are employed for determining the mean angular deviation, but for this experiment the co-ordinate method of Fowler[6] is used. This method uses a series of co-ordinate observations at equally spaced points along a track with the angular deviations being deduced from the second differences of the co-ordinates. Large deviations are discarded by a cut-off procedure which replaces any second difference by zero if it exceeds 4 times the mean of the absolute values of the second differences.

This multiple scattering techniques have been used to determine the energy of particles at low gamma and the results have been well verified. Assumptions made in these calculations are:

(1) The scattering constant, K, is known.

(2) The angular deviation or scatter in a certain cell length is a random variable.

(3) pv is constant over the distance of the scattering observations. This assumption is not correct, but for relatively short segments of track it is approximately true.

As the energy of a particle is increased it is necessary to use longer cell lengths to maintain a favorable signal to noise ratio. Signal is defined as the portion of the observed second differences which is caused by true scattering, and noise is the portion caused by errors introduced in the observation. However, long cell lengths are not feasible with high

energy electrons because assumption 3 will not be even approximately true over a distance comparable to a radiation length. Thus short cell lengths must be used to get sufficient data, i.e. a sufficient number of cells, before the probability of appreciable energy loss becomes high. The problem is to find the means to overcome the adverse signal to noise ratios which go with the short cell lengths and get meaningful results.

The scattering measurements were made on a Koristka R-4 microscope equipped with an eyepiece filar micrometer whose smallest division is 0.043 microns. Our measurements were estimated to a tenth of a division or 0.0043 microns. Measurements made on a single grain in the emulsion are reproducible within ± 0.02 microns, so the readout capabilities of the microscope are not a limiting factor in this experiment. However the Koristka is an extremely sensitive instrument and several precautions were taken to avoid introducing errors via the microscope. An enclosure 5 feet square and 7 feet high of medium weight cloth was made to protect the microscope and scanner from drafts which might cause temperature changes in the microscope structure and introduce distortions in the measurements. Also a minimum warm-up period or 30 minutes was allowed preceeding data taking, and once started, data was taken continuously for each track to minimize any microscope drift. Considerable care was taken to avoid backlash in the microscope movements.

An effort was made to determine the minimum signal that each scanner could detect. This was done by scattering flat tracks of 16.2 Bev π^- for a total of 3 to 5 cm per scanner at each of three cell lengths, 100, 250, and 500 microns. The theoretical rms signal from these cell lengths is 0.003, 0.013, and 0.042 microns respectively. We assume that observed second difference and signal are related by a quadratic function,

$$\overline{X^2} = D^2 + 6\,\overline{E^2}$$

where X is the observed second difference, D is the signal and $\sqrt{6}\ E$ is the noise. (See page 10 for an explanation of this equation). The observed second differences in this case were 0.11± 0.02, 0.14 ± 0.02, and 0.15 ± 0.02 microns respectively, for the 3 cell lengths. It is obvious that X is approximately equal to $\sqrt{6}\ E$, that the signal from the

16.2 Bev π^- is below the detection threshold, and that any second difference detected may be regarded as the minimum noise or "personal" noise for the scanner making the observations. These figures indicate that the personal noise is changing very little with cell length as compared to signal, which varies as $s^{3/2}$ where s is defined as cell length.

All the electron scattering data were taken from a single plate and the scattering was done by 3 different observers. Data were stored on punched cards for later analysis with the aid of a CDC 1604 computer.

Periodically, as a consistency check, the same track was scattered by each of the three observers. Energy determinations from these observations were within expected statistical fluctuations, which indicates that there are no systematic differences between scanners.

4. Methods of data reduction

A. Separation of Signal and Noise

The usual procedure followed in determining energy by means of multiple scattering measurements is to use the well known scattering formula

$$pv = \frac{K s^{\frac{3}{2}}}{573 D}$$

where K is the scattering constant[7], and s is the cell length in microns, where a cell is a periodic distance along the microscope's X axis. A method of finding D which begins by deriving the observed second difference on the Kth cell, X_k, in terms of seven independent variables has been outlined by Barkas[8]. X_k is the algebraic value of $Y_k - 2Y_{k+1} + Y_{k+2}$, where the Y's are ordinates measured from a straight line parallel to the particle path. In practice, this line is taken as the X motion of the microscope stage.

$$X_k = A_{k+1} + B_{k+1} + A_k - B_k + E_k + E_{k+2} - 2E_{k+1}$$

The variables are defined as follows:

$$A_{k+1} \equiv \frac{1}{2} (I_{k+1} + J_{k+1})$$

$$B_{k+1} \equiv \frac{1}{2} (I_{k+1} - J_{k+1})$$

where I_{k+1} and J_{k+1} are statistical variables which vary from cell to cell and are defined by the quantities.

$$I_{k+1} \equiv \sum_{i=1}^{n_{k+1}} \lambda_i \sum_{\gamma=1}^{i} W_\gamma \qquad J_{k+1} \equiv \sum_{j=1}^{n_{k+1}} W_j \sum_{\ell=1}^{j} \lambda_{\ell-1}$$

where W_i is the projected angle associated with the ith scatter, and λ_i is the particle's path length between the ith and the (i + 1)st scattering event in the (k + 1)st cell. These quantitites are displayed in Figure 3 and Figure 4.

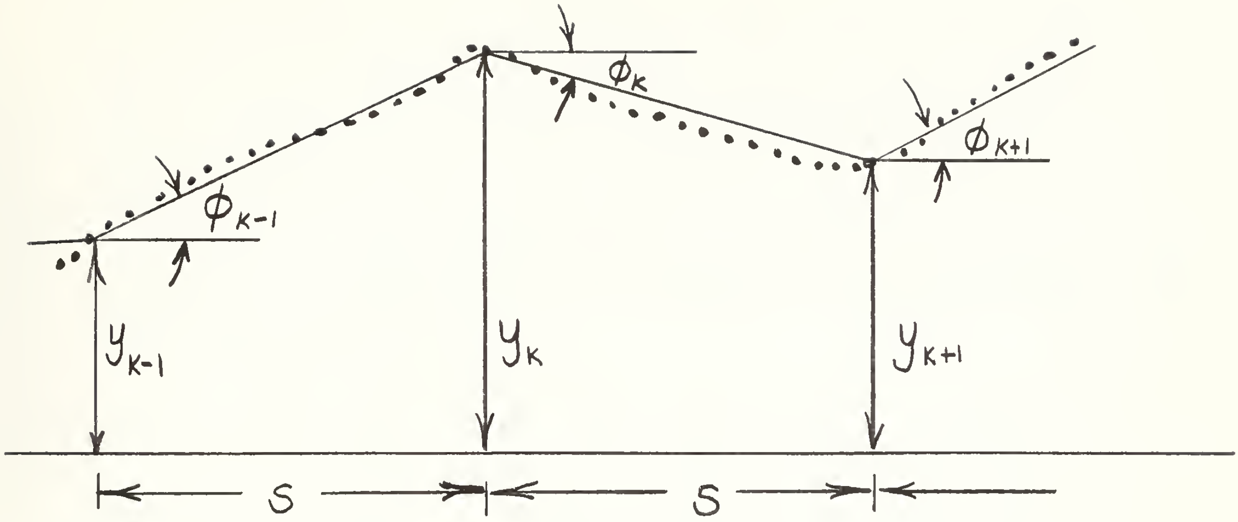


Fig. 3. Particle Track Showing Multiple Scattering

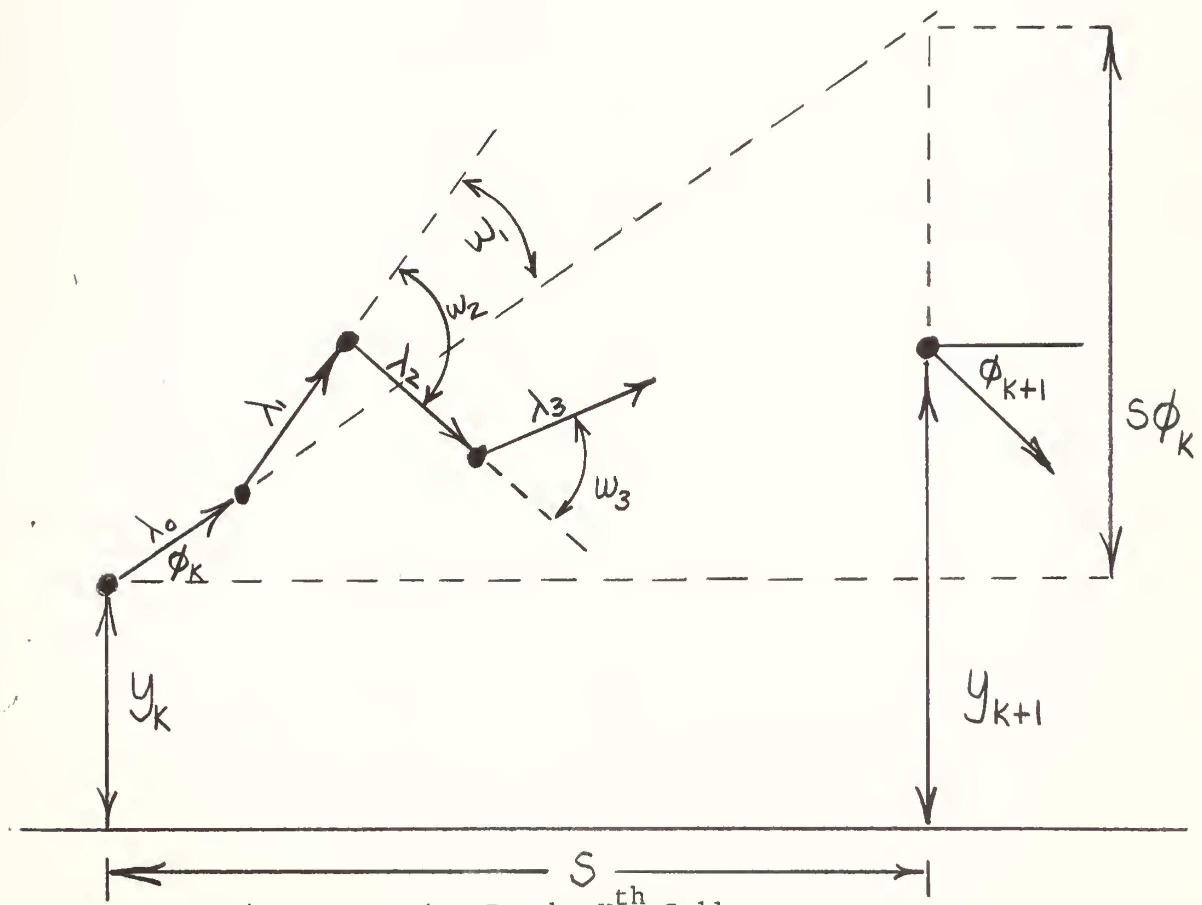


Fig. 4. Scattering In the Kth Cell

Similarly: $A_k = \frac{1}{2}(I_k + J_k)$ and $B_k = \frac{1}{2}(I_k - J_k)$

The remainder of the independent terms, E_k, E_{k+1}, and E_{k+2}, are the "noise errors" in the K^{th}, (k+1)st, and (k+2)nd ordinates. "Noise error" is the difference between the observed ordinate and the ordinate we would observe if there were no microscope error, scanner error, emulsion distortion, etc.

If we now form the mean value of the products $X_k K_k$, all the cross terms fall out, because they have an expectation value of zero. The remaining terms are:

$$\overline{X_k X_k} = \overline{A_{k+1}^2} + \overline{B_{k+1}^2} + \overline{A_k^2} + \overline{B_k^2} + \overline{E_k^2} + \overline{E_{k+2}^2} + \overline{4E_{k+1}^2}$$

Since noise is a random variable which is equally - likely on each measurement, the mean value of the squared terms maybe collected and treated simply as E^2. Two other useful relations, [8]

$$\overline{A_k^2} = 3\overline{B_k^2} \quad \text{and} \quad 8\overline{B_k^2} = D^2$$

may also be substituted into the $\overline{X_k X_k}$ equation so that: $\overline{X_k X_k} = D^2 + 6\overline{E^2}$

Similar development for other products are of the form $\overline{X_i X_j} = aD^2 + b\overline{E^2}$

Of particular concern are the following:

$$\overline{X_i X_i} = D^2 + 6\overline{E^2}$$

$$\overline{X_i X_{i+1}} = D^2/4 + 4\overline{E^2}$$

$$\overline{X_i X_{i+2}} = \overline{E^2}$$

$$\overline{X_i X_{i+3}} = 0$$

Combinations of the above, or other products so derived, may then be used to eliminate either noise or signal from a track. For example:

$$\frac{11}{8} D^2 = (\overline{X_i X_i} + \frac{3}{2} \overline{X_i X_{i+1}})$$

$$22E^2 = (\overline{X_i X_i} - 4\overline{X_i X_{i+1}})$$

Similar combinations may be formed from other products derived from second, third, or higher order differences. We do not consider third differences in this paper, however.

Another way of computing signal utilizes the method of variation of cell lengths. A study of this method has been done by di Corato, Hirschberg, and Locatelli[9]. Our development is similar although we include derivation of the exponent 3+Z used in the formula for ${}_nD^2$; a result not reported in the literature. By the symbol ${}_nD^2$ we mean the signal for a cell length ns, where n is on integer. Briefly, the development proceeds as follows:

The scattering constant appropriate for cell length ns derived by Scott[7] when disregarding any X_i which exceeds four times the mean of the absolute value of the X_i is

$${}_nK^2_{co} = 675 \left[0.090 + 0.272 \log_{10} (5ns)\right]$$

where n is an integer and s is the cell length in microns.

Now if we define ns' such that

$$ns' = ns/(0.23 + 0.77\beta^2) \text{ where } s' \longrightarrow s \text{ for } \beta \longrightarrow 1 \quad \text{then}$$

$${}_nK_{co} = {}_nK_{co} (\beta, ns)$$

If we now assume that E is independent of cell length and ignore the dependence of ${}_nK_{co}$ on n, we can write ${}_nX^2 = {}_nD^2 + E^2$

but $${}_nD^2 = \frac{K_{co}^2 (ns)^3}{(573)^2 (p\beta c)^2} = \frac{K_{co}^2 (ns)^3}{a}$$

so that $${}_nX^2 - E^2 = \frac{K_{co}^2 (ns)^3}{a}$$

Now we can remove noise by using measurements using two different cell lengths, ns and ms, where $n > m$.

$${}_nX^2 - {}_mX^2 = \frac{K_{co}^2}{a}\left[(ns)^3 - (ms)^3\right] = \frac{K_{co}^2 (ms)^3}{a}\left[(\frac{n}{m})^3 - 1\right] = \left[{}_mD^2 \; (\frac{n}{m})^3 - 1\right]$$

consequently

$${}_mD^2 \cong \frac{{}_nX^2 - {}_mX^2}{\left[(\frac{n}{m})^3 - 1\right]} \tag{4-1}$$

We now include the dependence of ${}_nK_{co}$ on n:

$${}_nX^2 - {}_mX^2 = \frac{1}{a}\left[{}_nK_{co}^2 \; (ns)^3 - {}_mK_{co}^2 (ms)^3\right] \tag{4-2}$$

but

$${}_nK_{co}^2 = 675 \left[0.09 + 0.18 \;\ell n \; (5ns)\right]$$

and

$$\frac{{}_nK_{co}^2}{{}_mK_{co}^2} \approx \frac{\ell n(5ns)}{\ell n(5ms)}$$

Then (4-2) becomes: $${}_nX^2 - {}_mX^2 = \frac{{}_mK_{co}^2 (ms)^3}{a} \left[\frac{\ell n \; 5ns}{\ell n \; 5ms} (\frac{n}{m})^3 - 1\right] \tag{4-3}$$

Now writing $\frac{\ell n\ 5ns}{\ell n\ 5ms} = (\frac{n}{m})^z$ and solving for z: $z = \frac{\ell n\left[\frac{\ell n\ 5ns}{\ell n\ 5ms}\right]}{\ell n\ (\frac{n}{m})} \approx \frac{1}{\ell n\ 5s}$

Where we have used $\ell n(1+z) \cong z$ for small z.

If we now assume that noise depends on cell length in such a way that noise squared varies as cell length to some power k so that ${}_nE^2 = b\ (ns)^k$

then

$${}_nX^2 - {}_nE^2 = \frac{{}_nK_{co}^2\ (ns)^3}{a}$$

$${}_mX^2 - {}_mE^2 = \frac{{}_mK_{co}^2\ (ms)^3}{a}$$

$$\frac{{}_nE^2}{{}_mE^2} = (\frac{n}{m})^k$$

so that

$${}_nX^2 - (\frac{n}{m})^k\ {}_mX^2 = \frac{{}_nK_{co}^2\ (ns)^3}{a} - (\frac{n}{m})^k\ \frac{{}_mK_{co}^2\ (ms)^3}{a} \qquad (4\text{-}4)$$

but since

$$\frac{{}_nK_{co}^2}{{}_mK_{co}^2} \approx (\frac{n}{m})^z$$

Equation (4-4) becomes:

$${}_nX^2 - (\frac{n}{m})^k\ {}_mX^2 = {}_mD^2\left[(\frac{n}{m})^{3+z} - (\frac{n}{m})^k\right]$$

or finally:

$${}_mD^2 = \frac{{}_nX^2 - (\frac{n}{m})^k\ {}_mX^2}{(\frac{n}{m})^{3+z} - (\frac{n}{m})^k}$$

For s = 100 microns and 200 microns; z equals 0.160 and 0.141. In our work we have used cells of 100 and 200 microns and a value of 0.15 for z. Also, our scattering of 16.2 Bev pions shows that noise does not depend appreciably on cell length so we have used k = 0. The final result is:

$$ {}_mD^2 = \frac{\dfrac{{}_nX^2 - {}_mX^2}{3.15}}{\left(\frac{n}{m}\right) - 1} $$

B. Statistical errors

In Section A of this subhead we developed difference product and cell overlap methods for determining signal and noise. Now we calculate the statistical error on these quantities in order to set statistical limits on the computed energies. To do this, we proceed in the following manner:

(a) form a combination to get D or E using overlap or difference-product methods

(b) write the X_i's in terms of the independent variables A, B, and E and find the mean value

(c) sum the variances of the independent variables

(d) compute the standard deviation from the variance using

$$ \sigma_A^2 = \overline{A^2} - \bar{A}^2 $$

For example: $\overline{X_i X_i} = D^2 + 6\overline{E^2}$

$$ \overline{X_i X_{i+1}} = \frac{1}{4} D^2 - 4\overline{E^2} $$

Define s such that

$$ s \equiv \sum_i^n \left(X_i X_i + \frac{3}{2} X_i X_{i+1}\right) = \sum_i^n \frac{11}{8} D^2 $$

then

$$ \bar{s} = n \frac{11}{8} D^2 $$

Now consider

$$ s_i = X_i X_i + \frac{3}{2} X_i X_{i+1} $$

Then $s_i = (A_{i+1} + A_i + B_{i+1} - B_i + E_i - 2E_{i+1} + E_{i+1})^2 + \frac{3}{2} (A_{i+1} + A_i$

$+ B_{i+1} - B_i + E_i - 2E_{i+1} + E_{i+1})(A_{i+2} + A_{i+1} + B_{i+2} - B_{i+1}$

$+ E_{i+1} - 2E_{i+2} + E_{i+2})$

The contribution from a single cell is reflected in 17 terms for the $X_i X_i$ product and 22 terms for the $X_i X_{i+1}$ product. However, s_i is composed of only 23 terms because the noise terms add to zero.

Forming the mean value of s_i we have

$$\overline{s}_i = \frac{7}{2} \overline{A_i^2} + \frac{1}{2} \overline{B_i^2}$$

where all the cross terms have vanished. But

$$\overline{A_i^2} = 3\overline{B_i^2} \text{ so that } \overline{s_i} = \frac{11}{3} \overline{A_i^2} \text{ and } \overline{s}_i^{\,2} = \frac{121}{9} (\overline{A_i^2})^2$$

In order to find $\overline{s_i^2}$, we form s_i^2 and take the mean value of each of the resulting terms. The first operation is tedious, giving 549 terms, but again all cross terms drop out when taking the mean value. The result is:

$$\overline{s_i^2} = \frac{49}{4} \overline{A_i^4} + \frac{1}{4} \overline{A_i^4} + \frac{56}{3} \overline{A_i^2}\, \overline{E^2} + \frac{286}{9} (\overline{A_i^2})^2 + \frac{42}{2} (\overline{E^2})^2$$

again $\overline{A_i^2} = 3\, \overline{B_i^2}$ and $\overline{A_i^4} = 3\, (\overline{A_i^2})^2$

so that

$$\sigma_{s_i}^2 = \overline{s_i^2} - \overline{s}_i^{\,2}$$

$$\sigma_{s_i}^2 = \frac{331}{6} (\overline{A_i^2})^2 + \frac{56}{3} \overline{A_i^2}\, \overline{E^2} + \frac{49}{2} (\overline{E^2})^2$$

Now we form $\dfrac{\sigma_{s_i}^2}{\overline{s}_i^2}$ and then $\dfrac{\sigma_s^2}{\overline{s}^2} \equiv \dfrac{1}{n} \dfrac{\sigma_{s_i}^2}{\overline{s}_i^2}$ is

$$\frac{\sigma_s^2}{\overline{s}^2} = \frac{1}{n}\left[4.08 + 1.39\frac{\overline{E^2}}{\overline{A_i^2}} + 1.82\frac{(\overline{E^2})^2}{(\overline{A_i^2})^2}\right]$$

where n is the number of independent terms in s.

but $\overline{s} = n\frac{11}{3}\overline{A_i^2}$ and $\overline{A_i^2} = \frac{3}{8}D^2$ so that $\overline{s} = n\frac{11}{8}D^2$

Now when D is the rms signal, $\dfrac{\sigma_s}{\overline{s}} = \dfrac{2\sigma_D}{D}$

so that $$\sigma_D = \frac{D}{\sqrt{n}}\left[1.02 + 0.926\frac{E^2}{D_2} + 3.23\frac{(\overline{E^2})^2}{(D^2)^2}\right]^{1/2}$$

We have extended this method of error analysis to energy determinations by the overlap method. Our first overlap combination uses unit and double cell lengths. Recalling the overlap formula

$$_mD^2 = \frac{\overline{_nX^2} - \overline{_mX^2}}{(\frac{n}{m})^{3+z} - 1}$$

we let n = 2, m = 1, and z = 0.15. Then $D^2 = \dfrac{\overline{_2X^2} - \overline{_1X^2}}{2^{3.15} - 1}$

Now form $V_i = \sum(_2X_i)^2 - (_1X_i)^2 \qquad = \sum(X_{i+2} + 2X_{i+1} + X_i)^2 - (X_i)^2$

Once again the second differences can be written in terms of A's, B's, and E's and the mean value taken. The result is:

$$\overline{V_i} = 18\,\overline{A_i^2} + 2\overline{B_i^2} \qquad \text{or} \qquad \overline{V_i} = 7D^2$$

Similarly $$V_i^2 = \sum^n \left[({}_2X_i)^2 - (X_i)^2 \right]^2$$

gives $$\overline{V_i^2} = \frac{6292}{3} (\overline{A_i^2})^2 + \frac{768}{3} \overline{A_i^2}\,\overline{E^2} + 168(\overline{E^2})^2$$

Now returning to our original expressions for V_i

$$V_i = \sum^n ({}_2X_i)^2 - (X_i)^2$$

$$\overline{V_i} = n \left[(\overline{{}_2X_i})^2 - \overline{X_i^2} \right]$$

$$\overline{V_i} = n \left[2^{3.15} - 1 \right] D^2$$

but $$D^2 = \frac{8}{3} \overline{A_i^2}$$

so that $$\sigma_{V_i}^2 = \overline{V_i^2} - (\overline{V_i})^2$$

$$\sigma_{V_i}^2 = \frac{15740}{9} (\overline{A_i^2})^2 + \frac{768}{3} \overline{A_i^2}\;\overline{E^2} + 168\,(\overline{E^2})^2$$

but $$D^2 = \frac{8}{3} \overline{A_i^2} \;;\quad \sigma_V^2 = n\,\sigma_{V_i}^2 \quad \text{and } \overline{V} = n\overline{V_i} = 7nD$$

Now $$\frac{\sigma_V}{\overline{V}} = \frac{2\sigma_D}{D} \quad \text{so that } \sigma_D = \frac{D}{\sqrt{n}} \left[1.25 + 0.49\,\frac{\overline{E^2}}{D^2} + 0.32\,\frac{(\overline{E^2})^2}{(D^2)^2} \right]^{1/2}$$

In summary, we have developed the following formulae from the relationships indicated. The formulae are grouped in 4 sets called MAGIC 1, 2, 3, 4 for fortran coding purposes. Each MAGIC provide a unique way to determine track characteristics and particle energies.

MAGIC 1 uses difference products

$$s = \sum (X_i X_i + \frac{3}{2} X_i X_{i+1})$$

$$\bar{s} = n \frac{11}{8} D^2$$

$$\frac{\sigma_D}{D} = \frac{1}{\sqrt{n}} \left[1.02 + 0.926 \frac{\overline{E^2}}{D^2} + 3.23 \frac{(\overline{E^2})^2}{(D^2)^2} \right]^{1/2}$$

$$F = \sum (X_i X_i - 4 X_i X_{i+1})$$

$$\bar{F} = n\, 22\, \overline{E^2}$$

$$\frac{\sigma_E}{\bar{E}} = \frac{1}{\sqrt{n}} \left[1.29 + 0.11 \frac{D^2}{\overline{E^2}} + 0.0074 \frac{(D^2)^2}{(\overline{E^2})^2} \right]^{1/2}$$

MAGIC 2 uses difference products

$$L = \sum (X_i X_i - 6 X_i X_{i+2})$$

$$\bar{L} = {}_n D^2$$

$$\frac{\sigma_D}{D} = \frac{1}{\sqrt{n}} \left[10.219 + 76.00 \frac{\overline{E^2}}{D^2} + 506.00 \frac{(\overline{E^2})^2}{(D^2)^2} \right]^{1/2}$$

$$G = \sum X_i X_{i+2}$$

$$\bar{G} = n\, \overline{E^2}$$

$$\frac{\sigma_E}{\bar{E}} = \frac{1}{\sqrt{n}} \left[17.75 + 2.00 \frac{D^2}{\overline{E^2}} + 0.28 \frac{(D^2)^2}{(\overline{E^2})^2} \right]^{1/2}$$

MAGIC 3 uses unit and double unit cells

$$Q = \sum ({}_{2}X_i^2 - X_i^2)$$

$$\bar{Q} = 7 \; {}_{n}D^2$$

$$\frac{\sigma_D}{D} = \frac{1}{\sqrt{n}} \left[1.25 + 0.49 \frac{\overline{E^2}}{D^2} + 0.32 \frac{(\overline{E^2})^2}{(D^2)^2} \right]^{1/2}$$

$$R = \sum \left[8 \, (X_i)^2 - ({}_{2}X_i)^2 \right]$$

$$\bar{R} = 42 \; n \; \overline{E^2}$$

$$\frac{\sigma_E}{\bar{E}} = \frac{1}{\sqrt{n}} \left[1.163 + 0.109 \frac{D^2}{\overline{E^2}} + 0.0109 \frac{(D^2)^2}{(\overline{E^2})^2} \right]^{1/2}$$

MAGIC 4 uses unit and triple unit cells

$$T = \sum \left[({}_{3}X_i)^2 - (X_i)^2 \right]$$

$$\bar{T} = 26 \; {}_{n}D^2$$

$$\frac{\sigma_D}{D} = \frac{1}{\sqrt{n}} \left[1.679 + 0.147 \frac{\overline{E^2}}{D^2} + 0.050 \frac{(\overline{E^2})^2}{(D^2)^2} \right]^{1/2}$$

$$U = \sum \left[27 (X_i)^2 - ({}_{3}X_i)^2 \right]$$

$$\bar{U} = 156 \; n \; \overline{E^2}$$

$$\frac{\sigma_E}{\bar{E}} = \frac{1}{\sqrt{n}} \left[1.01 + 0.111 \frac{D^2}{\overline{E^2}} + 0.023 \frac{(D^2)^2}{(\overline{E^2})^2} \right]^{1/2}$$

C. Mechanics of data reduction

The calculations associated with the data reduction were done by means of a computer program, Program NIRVANA, written for the CDC 1604 computer. The main features of the program are as follows:

(1) Four routines are included for pv calculations. Each routine calculates the rms signal and noise, their standard deviation and fractional standard deviation, and the signal to noise ratio. Two of the routines use the difference - product method and two use the multiple cell length approach as outlined in the previous section. Each track is analyzed by each routine.

(2) Another routine looks at the observed distribution of the second differences, compares it with a gaussian distribution, and computes several moments of the distribution.

(3) Other features are:

(a) Each track may be segmented if desired and each segment treated as a separate track. The tracks are also treated in their entirety in addition to any segmenting.

(b) A calculation of the mean value of $X_k X_{k+3}$ was done for each track. This correlation should give a result of zero. The observed result is used as a creditability check for the track.

(c) Errors for the computed energies are asymmetric. The asymmetry stems from the standard deviation on the signal, which appears in the denominator of the scattering formula.

(d) The difference-product routines compute track characteristics and makes energy determinations using cell multiplicity, M, of 1, 2 and 3 times the primary cell length.

In summary, track characteristics of each track and the resulting energy determinations are done by eight processes in addition to any segmenting. A simplified flow diagram and program listing appears in the appendix.

5. Track Simulation

Originally, it was desired to scatter single tracks for long distances (1-3 cm) and experimentally determine the bremsstrahlung effects on the energy calculation for these data. As the results from the early data were calculated, it was obvious that the energy was lower than expected by a factor of 2 to 3, even when the distance scattered was comparatively short, which precludes large energy losses caused by radiative effects. We suspected these large and apparently systematic departures from the known energy of the electrons were a result of the fact that the true signal of a very energetic particle in a short cell length is so small as to be of the same order of magnitude as the noise in the observed signal, and that this relatively unfavorable signal to noise ratio was hiding any information about radiative effects.

At this point the necessity of a better data reduction method or methods became obvious. It was decided that a comparison of the various data reduction methods and an investigation of just how noise affects the energy calculations should be the next step. Thus was born the idea for the track simulation procedure, which later bore out our suspicion that high noise will depress the calculated energy.

The track simulation procedure involved basically two steps, first the construction of a "fake" track and second, the application of noise in small increments to this track. The "fake" track is a noise free simulated electron track constructed by forming a set of ordinates, or Y_i's, from a gaussian distribution of second differences. The increments of noise were calculated from a gaussian distribution also, and were randomly added to the Y_i's.

The track was analyzed by Program NIRVANA in the noise free condition, and an increment of noise was added and the track was analyzed again, and so forth. Values of E ranged from 0 to 0.60 microns in increments of 0.04 microns. This covers the range of E values calculated for real tracks by NIRVANA.

A series of fake tracks was analyzed and the results were as previously noted, that increasing the noise lowers the calculated energy.

This energy depression is more severe in the simulated tracks of particles with high energy and less severe in the lower energy tracks, as shown graphically by Figures 5, 6 and 7. These tracks were analyzed by MAGIC 1 (M = 1,2,3), and MAGIC 3.

Fig. 5. The Effect of Adding Noise to a Simulated 300 Mev Track

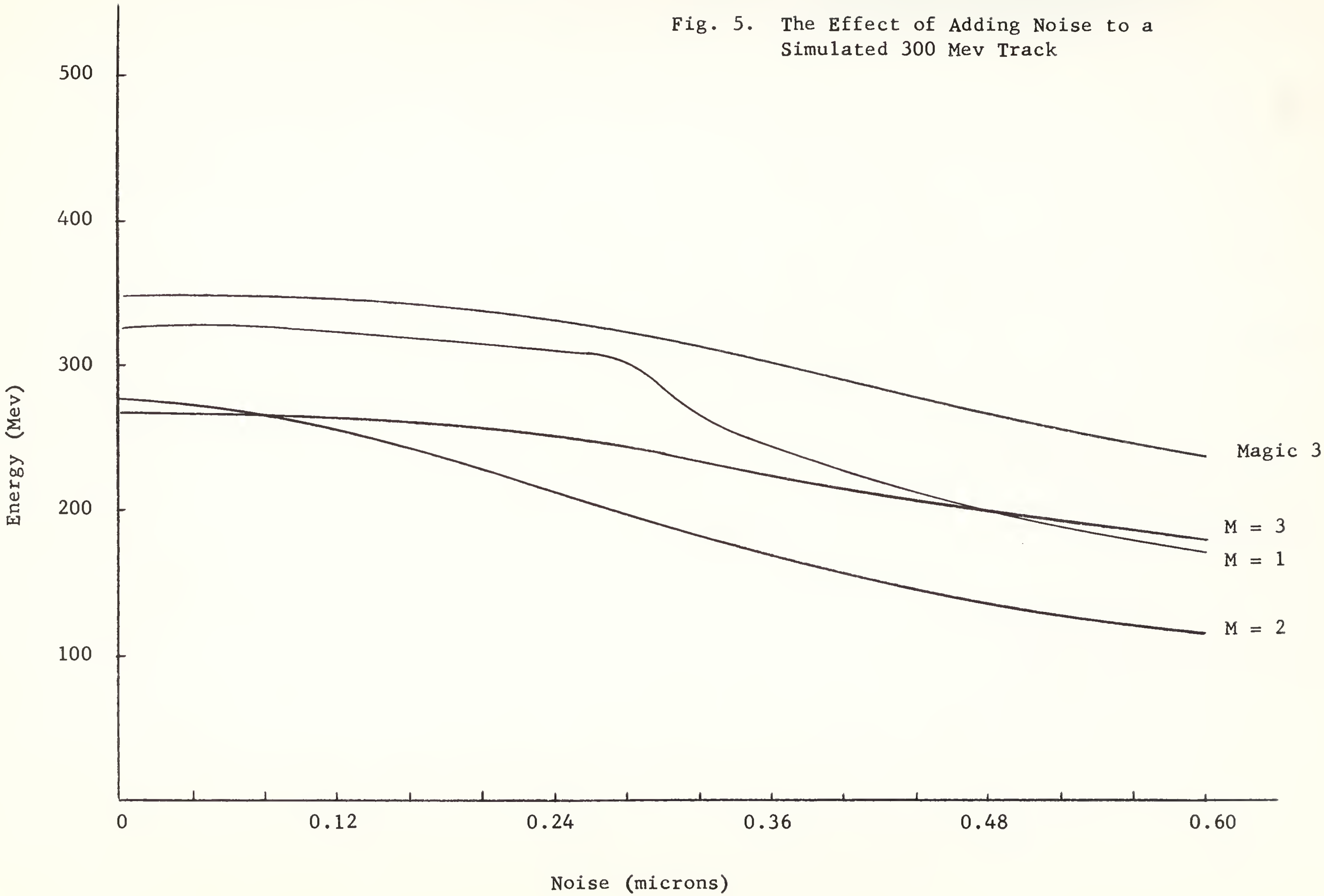


Fig. 6. The Effect of Adding Noise to a Simulated 500 Mev Track

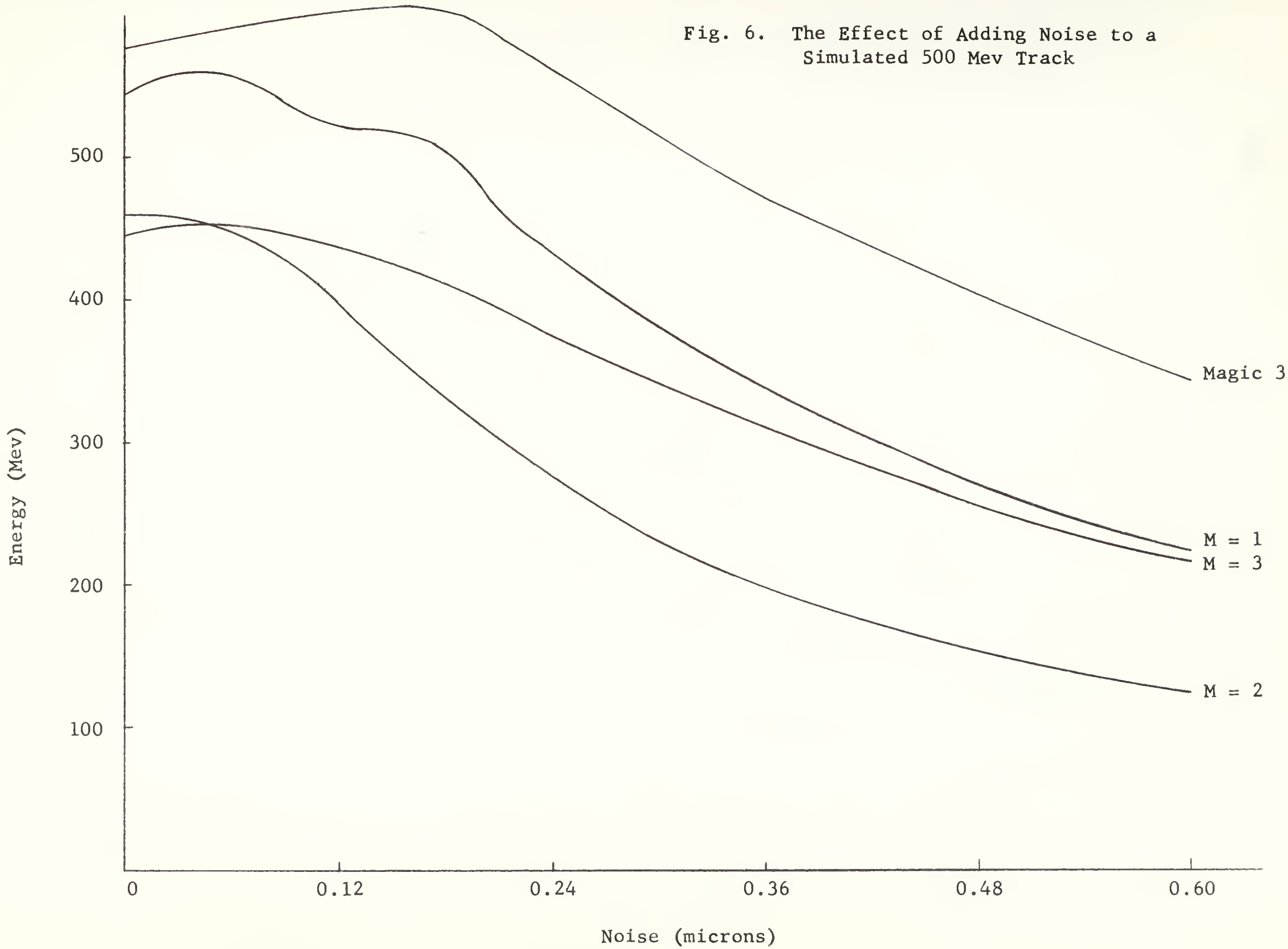
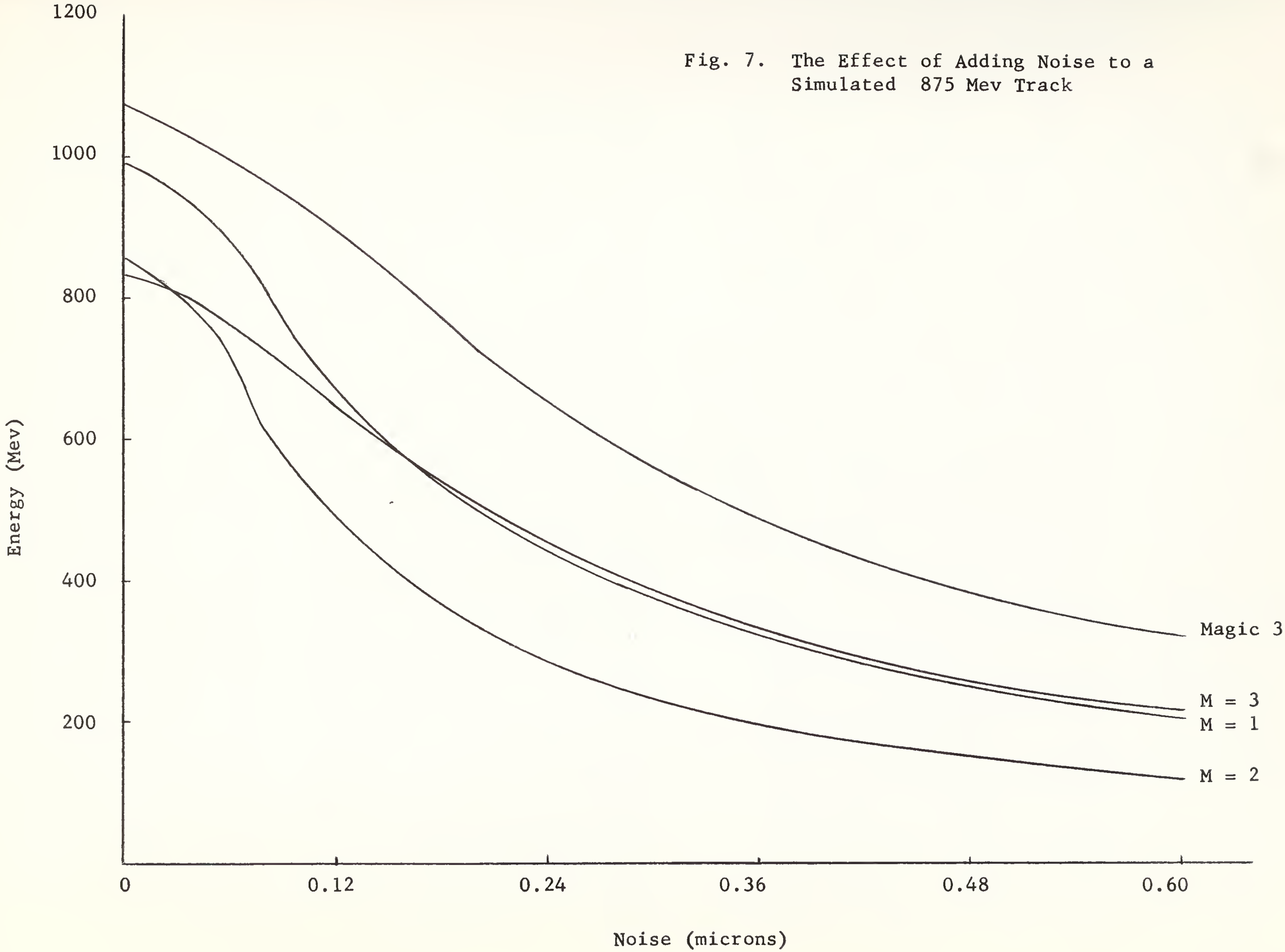


Fig. 7. The Effect of Adding Noise to a Simulated 875 Mev Track



6. Results

This experiment was designed to investigate the accuracy of the high energy points on the ionization curve, to experimentally determine the effects of radiation losses at high energy as a function of track length, and to compare the cell overlap and difference product methods of energy determination.

We found that our calculated values of energy were much lower than expected. The average values were low by factors that ranged from 1.3 for 300 Mev electrons to 2.73 for 875 Mev electrons.

The effects of radiative energy loss cannot be determined because the calculated energies were apparently degraded by other factors to a greater extent than could be attributed to radiation losses.

The difference product method and the cell overlap method give good results when the noise remains approximately constant with cell length, but all methods give low values for energy when the noise increases with cell length. The noise is calculated by Program NIRVANA from the input distribution of X_i's and the <u>known</u> <u>energy</u> using

$$6\overline{E^2} = \overline{X^2} - \overline{D^2}.$$

This value is the noise which must be subtracted from the observed second difference to get the correct value of D^2. Any method of data reduction which "detects" this amount of noise will therefore give the correct particle energy. Evidently, neither the product or overlap methods correctly evaluate the noise unless it is in fact independent of cell length.

There were some events in which the noise decreased with cell length, but all of these events had a low value for noise at M = 1[2], which usually went to zero at M = 2 and M = 3. For example, at 300 Mev 15% of the events were in this category. Because of the small change in the noise, these events were considered to be in the group with those of constant noise.

[2] M=1, 2, and 3 refer to cell multiplicity where M is the multiplier of the primary cell length.

The majority of the events had increasing values for noise with increasing cell length. For instance, about 70% of the 300 and 500 Mev events had values of noise that increased by a factor of more than 2 from M = 1, to M = 3, and 86 out of 90 events at 875 Mev fall in this group.

The data reduction routines assume constant noise, therefore, any increase in noise would be interpreted as an increase in signal. Thus it is not surprising that our values for calculated energies are low. The point in doubt is how to foretell which behavior the noise in a particular track will follow, i.e., whether it will increase or not. It is impressively obvious from our calculations that the previous theory of cell independent noise is not true at least 70% of the time for tracks of 300 Mev electrons and that this percentage grows rapidly to above 90% for 875 Mev electrons.

That the energy depression can also be caused by cell independent noise in the track being observed is demonstrated by our track simulation procedure. The effects of added noise at the different incoming energies are graphically displayed by Figures 5,6 and 7. The result that additional noise plays a larger role in depressing the calculated values of energy as the particle's initial energy is increased is to be expected because of the smaller total signal involved, but it is a vivid reminder of the possibilities for erroneous results at high gamma.

Our subroutine MAGIC 2 used correlations between $X_i X_i$ and $X_i X_{i+2}$ which proved to be a weak correlation, with a tremendous range in the answers. This does not appear in the averages which follow, but for this reason results from MAGIC 2 were not used in reaching our conclusions. A summary of our results is given below.

300 Mev Results

77.24 cm track scattered. Average energies, calculated from 162 tracks are:

	MAGIC 1		MAGIC 2
M = 1	202 Mev	M = 1	178 Mev
M = 2	240 Mev	M = 2	211 Mev
M = 3	255 Mev	M = 3	231 Mev

MAGIC 3: 218 Mev

MAGIC 4: 232 Mev

500 Mev Results

49.07 cm track scattered. Average energies calculated from 99 tracks are:

MAGIC 1		MAGIC 2	
M = 1	326 Mev	M = 1	191 Mev
M = 2	381 Mev	M = 2	241 Mev
M = 3	410 Mev	M = 3	274 Mev

MAGIC 3	MAGIC 4
352 Mev	381 Mev

875 Mev Results

36.62 cm track scattered. Average energies calculated from 90 tracks are:

MAGIC 1		MAGIC 2	
M = 1	320 Mev	M = 1	298 Mev
M = 2	353 Mev	M = 2	284 Mev
M = 3	384 Mev	M = 3	288 Mev

MAGIC 3	MAGIC 4
340 Mev	359 Mev

For those tracks in which noise appeared to be independent of cell length, we find:

For 36 300 Mev tracks,

MAGIC 1		MAGIC 2	
M = 1	265	M = 1	178
M = 2	356	M = 2	284
M = 3	394	M = 3	312

MAGIC 3	MAGIC 4
310	339

For 37 500 Mev tracks,

MAGIC 1		MAGIC 2	
M = 1	425	M = 1	225
M = 2	531	M = 2	331
M = 3	557	M = 3	348

MAGIC 3	MAGIC 4
481	529

For 4 875 Mev tracks,

MAGIC 1		MAGIC 2	
M = 1	411	M = 1	204
M = 2	700	M = 2	312
M = 3	800	M = 3	509

MAGIC 3	MAGIC 4
601	677

7. Critique

We list here some possible extensions of this work which are suggested by our data.

(1) Our analysis does not include correlations among third or higher order differences. These can readily be inserted into our program NIRVANA. It is possible that some set of correlations not yet tried will give more consistent results than those we have used.

(2) The track simulation proceedure should be improved. We have used a Gaussian distribution of second differences, which may well be an inadequate approximation to the true scattering distribution. Further, it would be of interest to introduce cell-dependent noise into simulated tracks and attempt to devise a way to treat it.

(3) We usually have to deal with tracks of only about 30 or 40 primary cells. More efficient estimates of the expectation values of various correlations for these small statistical samples may exist.

(4) Our data were taken from an emulsion plate of high random grain background and low track grain density. The effect of mistaking a background grain for a track grain should be investigated - this problem may have influenced our results considerably.

(5) Clearly, a method to deduce from observed distributions whether or not noise is cell-dependent, and then to adjust the data reduction routines to allow for it, must be developed.

APPENDIX I

SCATTERING PROGRAM

Program NIRVANA was written as a collection of subroutines in Fortran 60 for use exclusively on the CDC 1604 computer at US Naval Postgraduate School. A check for end of data would have to be inserted in the program for use on other computers; this detail is taken care of by the input routine of the computer at the US Naval Postgraduate School facility.

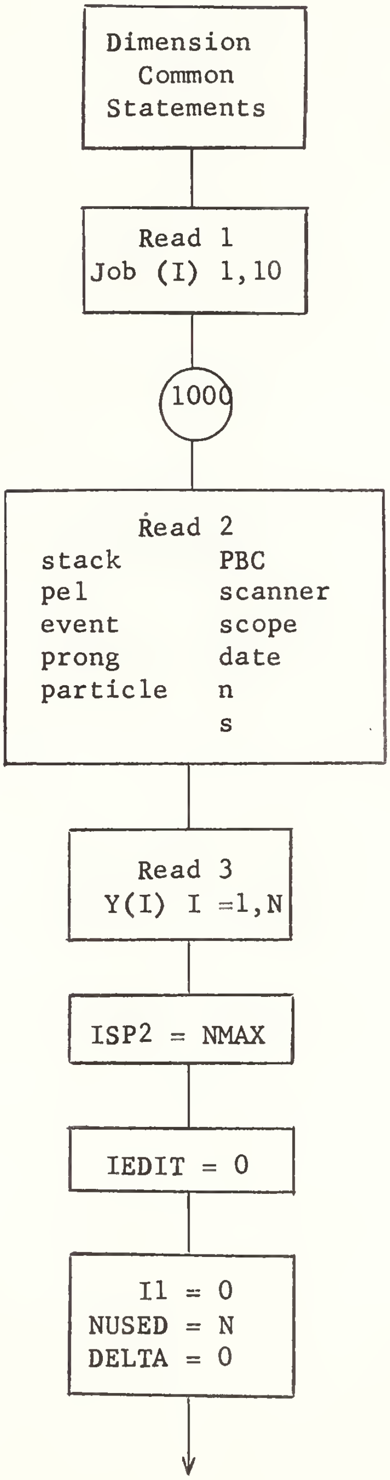
The program is a working program, that is, computer efficiency has been sacrificed when necessary to retain flexibility in use of the program. As it is, NIRVANA's subroutines may be substituted easily and quickly to experiment using other difference-product correlations. The various subroutines may be called at will if specific calculations are desired rather than wasting computer time by calling every subroutine for each event.

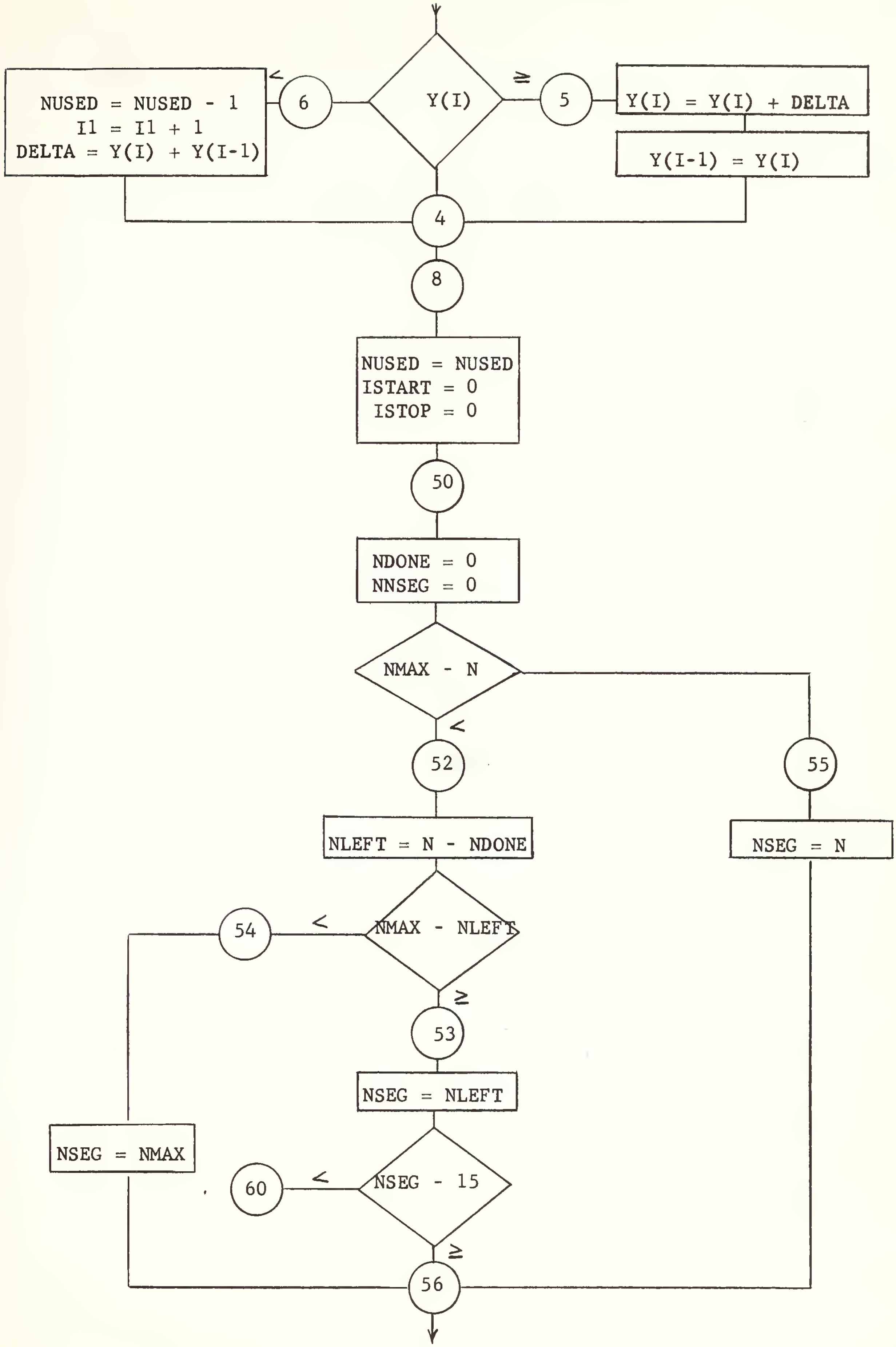
Comment cards have been used liberally throughout to aid in clarity. Symbols used in the program which are not obvious or have not been defined in a comment are defined below:

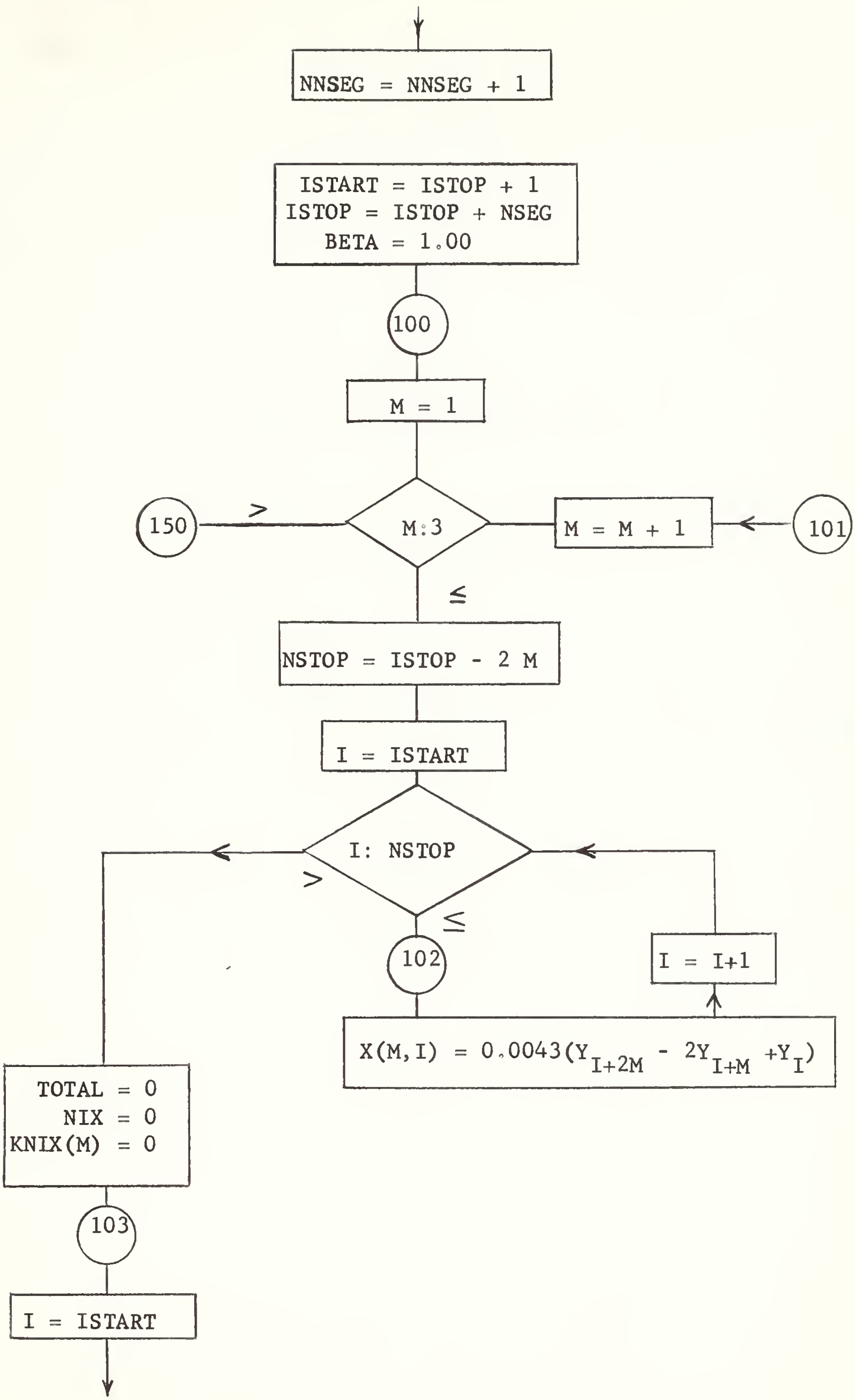
ISTACK	Emulsion stack identification
IPEL	Pellicle number
IEVENT	Event number
IPRONG	Prong number
IPTCL	Particle identification
PBC	Particle momentum times velocity, if known
ISCAN	Scanner identification
ISCOPE	Microscope identification
IDATE	Date
N	Number of Y_i's in the event
S	Cell length in microns

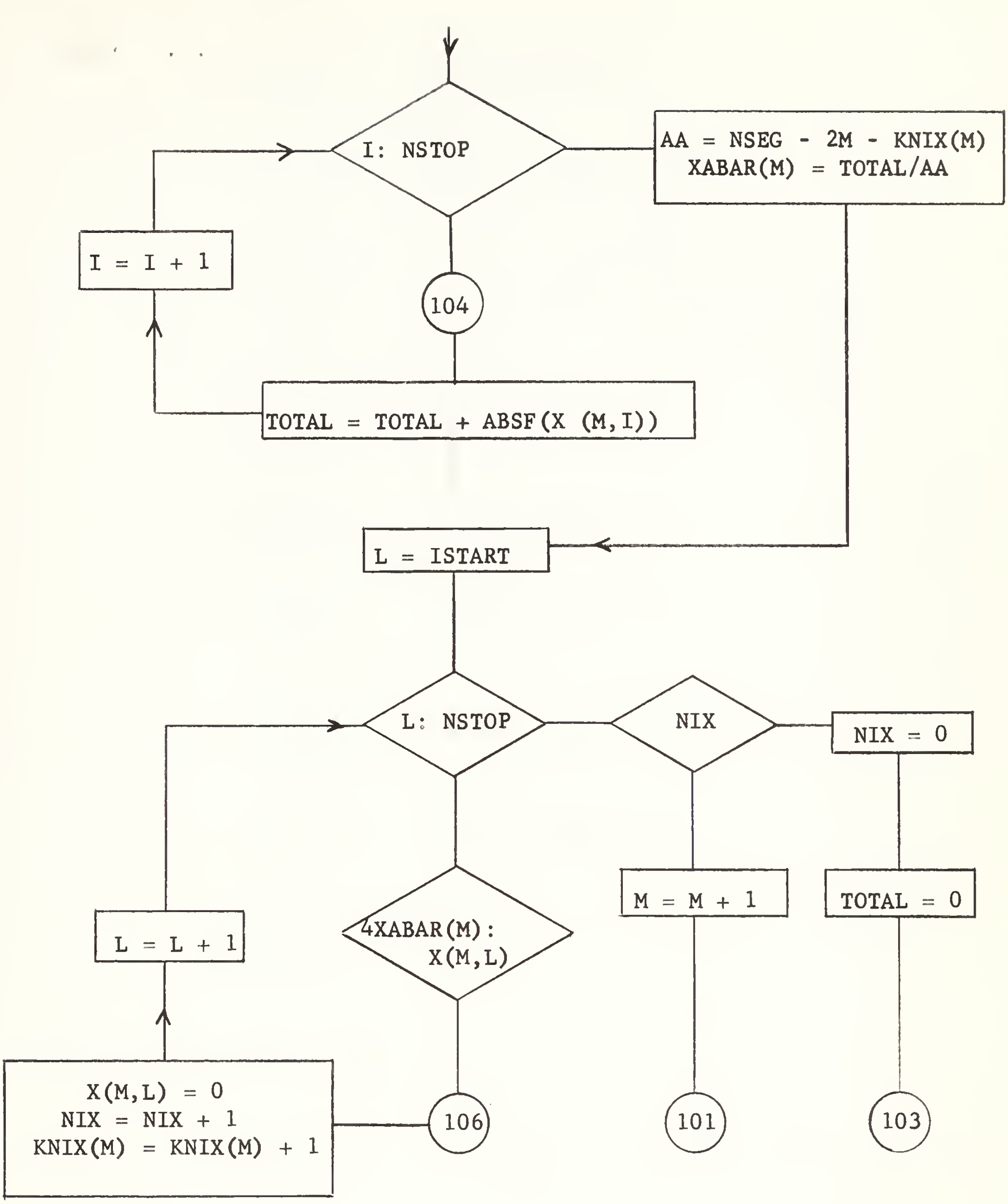
APPENDIX II

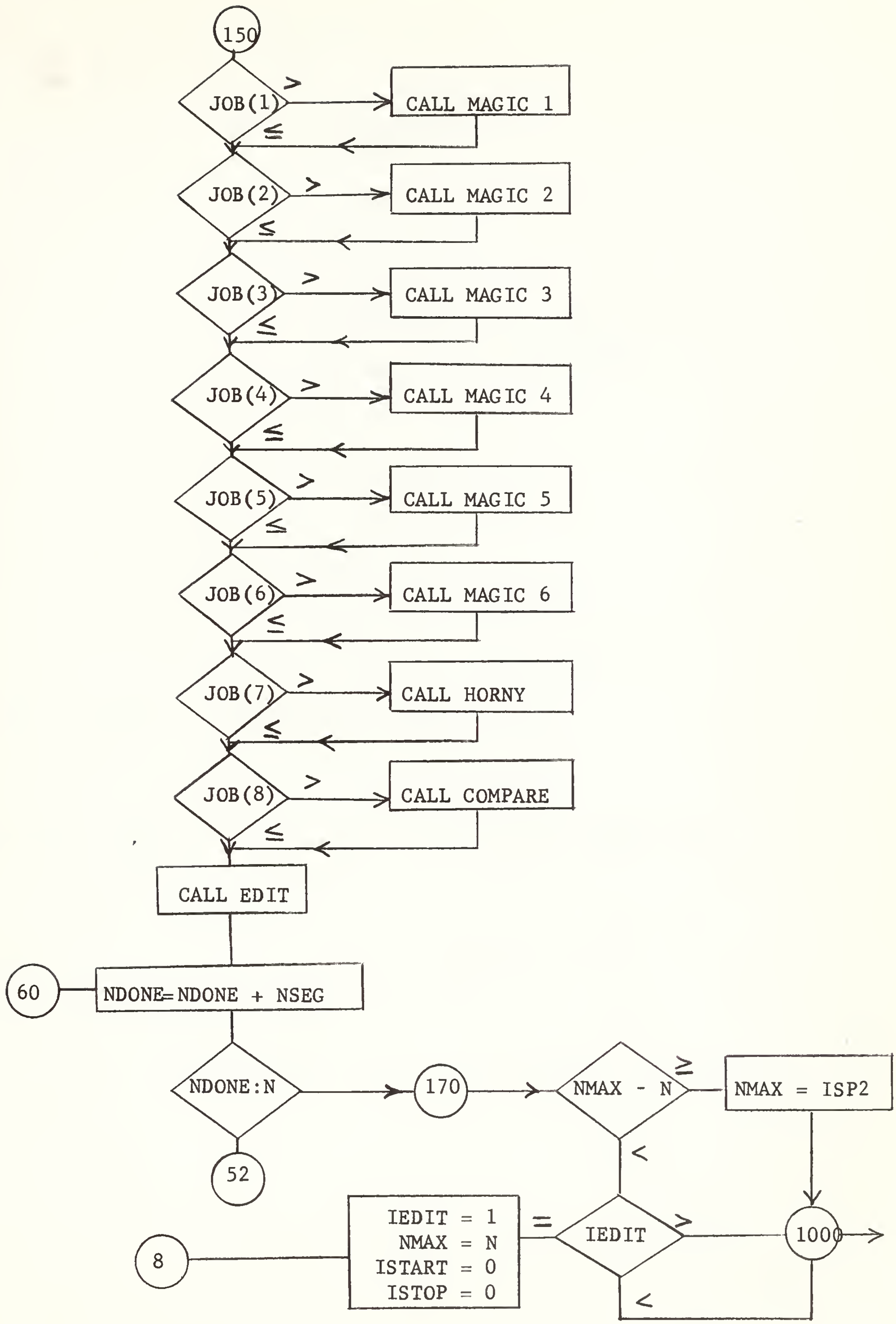
PROGRAM NIRVANA FLOW CHART



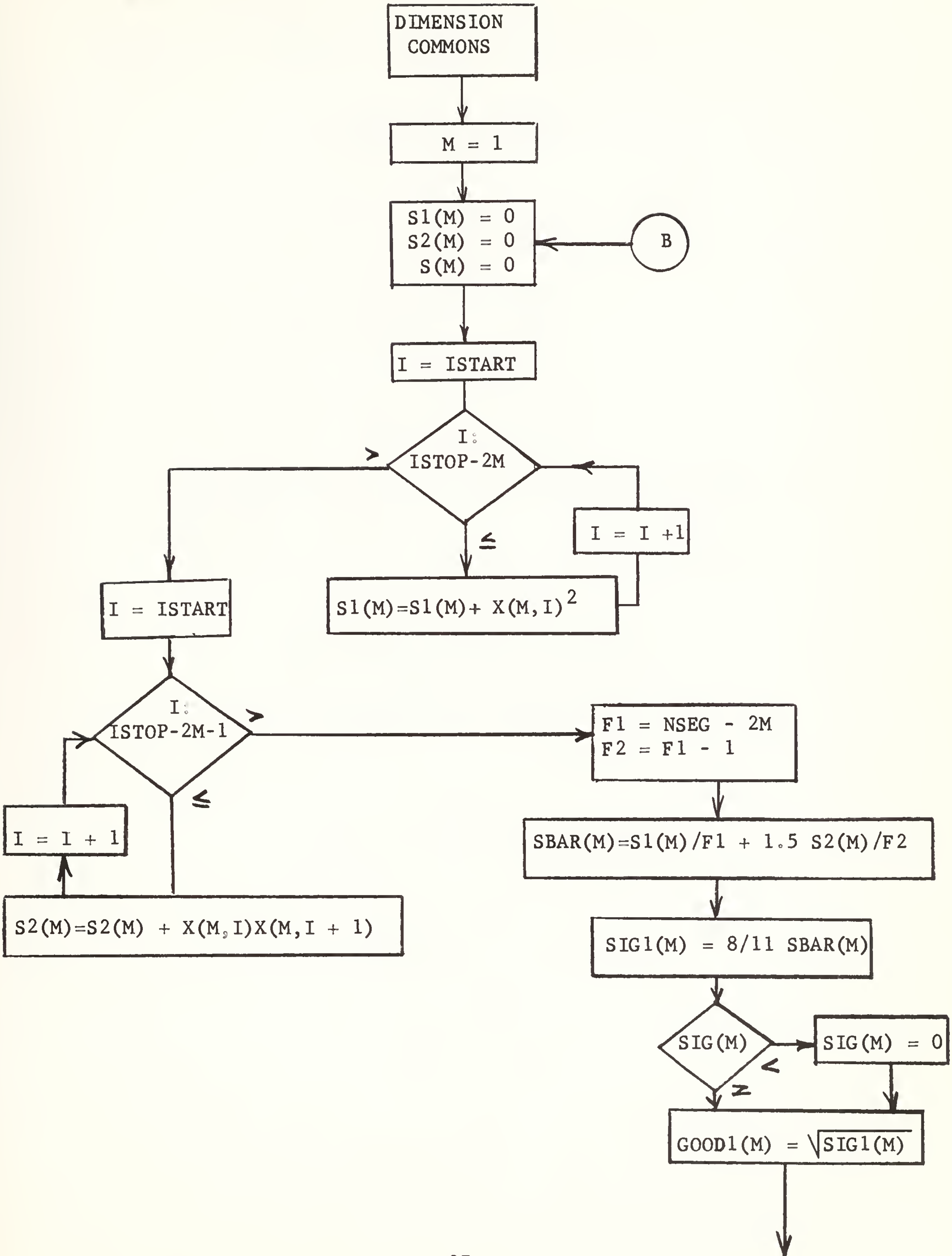

Y(I)
<
6
NUSED = NUSED - 1
I1 = I1 + 1
DELTA = Y(I) + Y(I-1)
≥
5
Y(I) = Y(I) + DELTA
Y(I-1) = Y(I)
4
8
NUSED = NUSED
ISTART = 0
ISTOP = 0
50
NDONE = 0
NNSEG = 0
NMAX - N
<
52
55
NLEFT = N - NDONE
NSEG = N
NMAX - NLEFT
<
54
≥
53
NSEG = NLEFT
NSEG = NMAX
NSEG - 15
<
60
≥
56


NNSEG = NNSEG + 1
ISTART = ISTOP + 1
ISTOP = ISTOP + NSEG
BETA = 1.00
100
M = 1
150
>
M:3
M = M + 1
101
≤
NSTOP = ISTOP - 2 M
I = ISTART
I: NSTOP
>
≤
102
I = I+1
X(M,I) = 0.0043(Y I+2M - 2Y I+M +Y I)
TOTAL = 0
NIX = 0
KNIX(M) = 0
103
I = ISTART

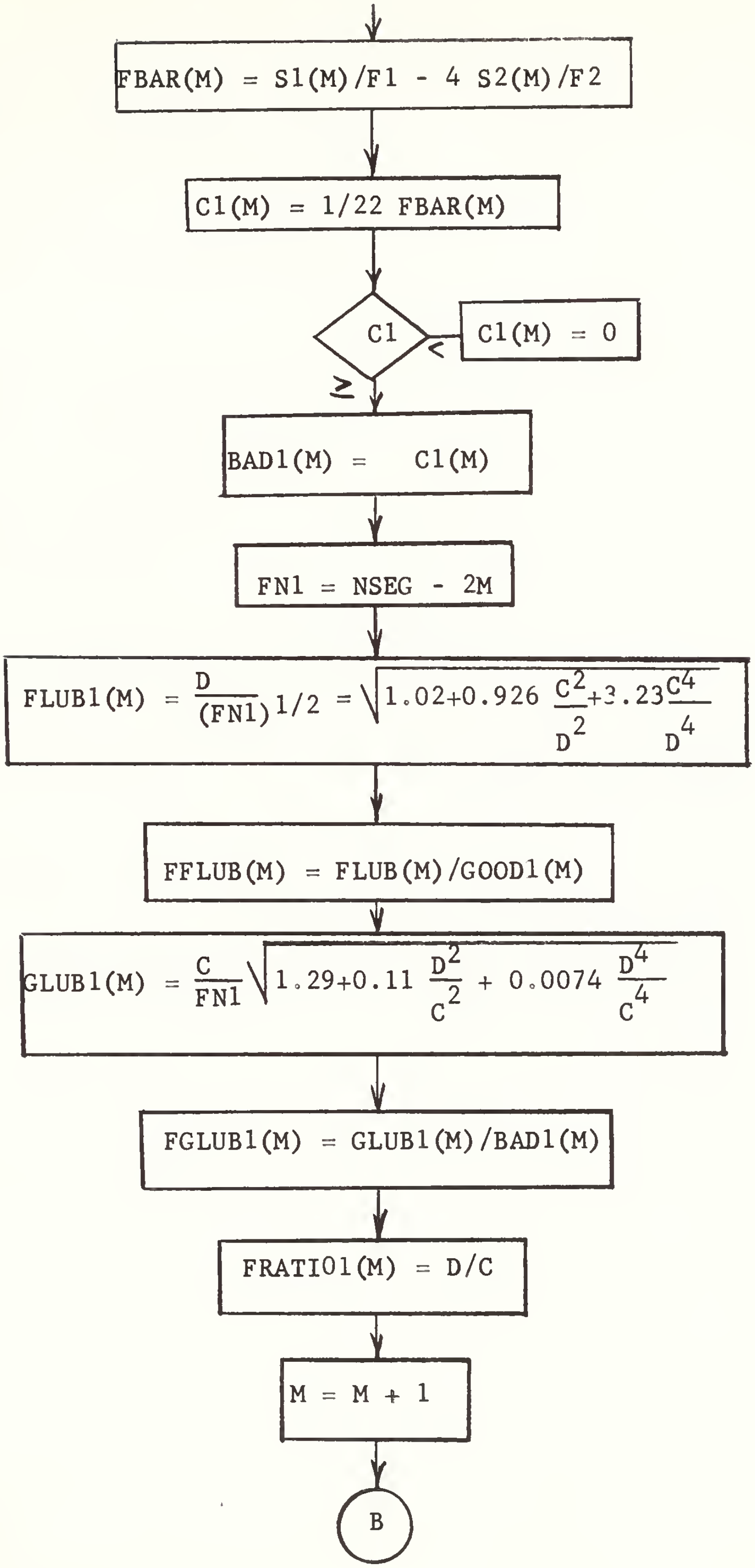

I: NSTOP
AA = NSEG - 2M - KNIX(M)
XABAR(M) = TOTAL/AA
I = I + 1
104
TOTAL = TOTAL + ABSF(X (M,I))
L = ISTART
L: NSTOP
NIX
NIX = 0
M = M + 1
TOTAL = 0
L = L + 1
4XABAR(M):
X(M,L)
X(M,L) = 0
NIX = NIX + 1
KNIX(M) = KNIX(M) + 1
106
101
103


150
JOB(1)
CALL MAGIC 1
JOB(2)
CALL MAGIC 2
JOB(3)
CALL MAGIC 3
JOB(4)
CALL MAGIC 4
JOB(5)
CALL MAGIC 5
JOB(6)
CALL MAGIC 6
JOB(7)
CALL HORNY
JOB(8)
CALL COMPARE
CALL EDIT
60
NDONE=NDONE + NSEG
NDONE:N
52
170
NMAX - N
NMAX = ISP2
8
IEDIT = 1
NMAX = N
ISTART = 0
ISTOP = 0
IEDIT
1000

SUBROUTINE MAGIC 1



SUBROUTINE MAGIC 1 (con't)



SUBROUTINE MAGIC 2

LOGIC SAME AS MAGIC 1

$$G1(M) = \sum X(M,I)^2$$

$$G2(M) = \sum X(M,I)X(M,I+2)$$

$$GBAR(M) = GE(M)/NSEG-2M-2$$

$$FLBAR(M) = G1(M)/NSEG-2M - 6\ G2(M)/NSEG - 2M - 2$$

$$SIG2(M) = FLBAR(M)$$

$$GOOD(M) = \sqrt{SIG2(M)}$$

$$C2(M) = GBAR(M)$$

$$BAD2(M) = \sqrt{C2(M)}$$

$$FLUB2(M) = \frac{mD}{\sqrt{n_2}} \sqrt{10.219 + 76.00 \frac{m\langle C^2\rangle}{mD^2} + 506 \frac{m\langle C^2\rangle^2}{mD^4}}$$

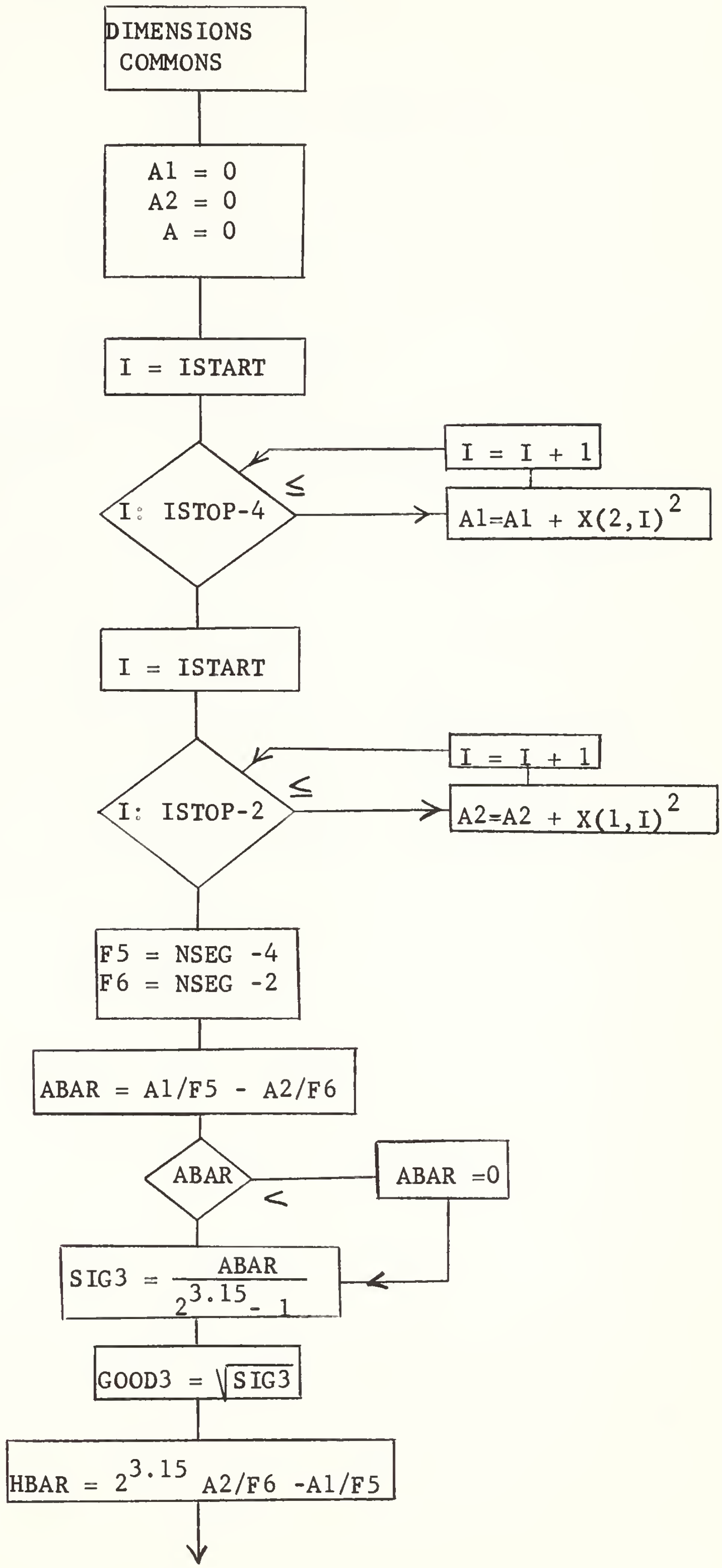
$$FFLUB2(M) = E/D$$

$$GLUB2(M) = \frac{\langle mC\rangle}{\sqrt{n_2}} \sqrt{17.75 + 2.0 \frac{mD^2}{\langle C^2\rangle} + 0.28 \frac{mD^4}{m\langle C^2\rangle^2}}$$

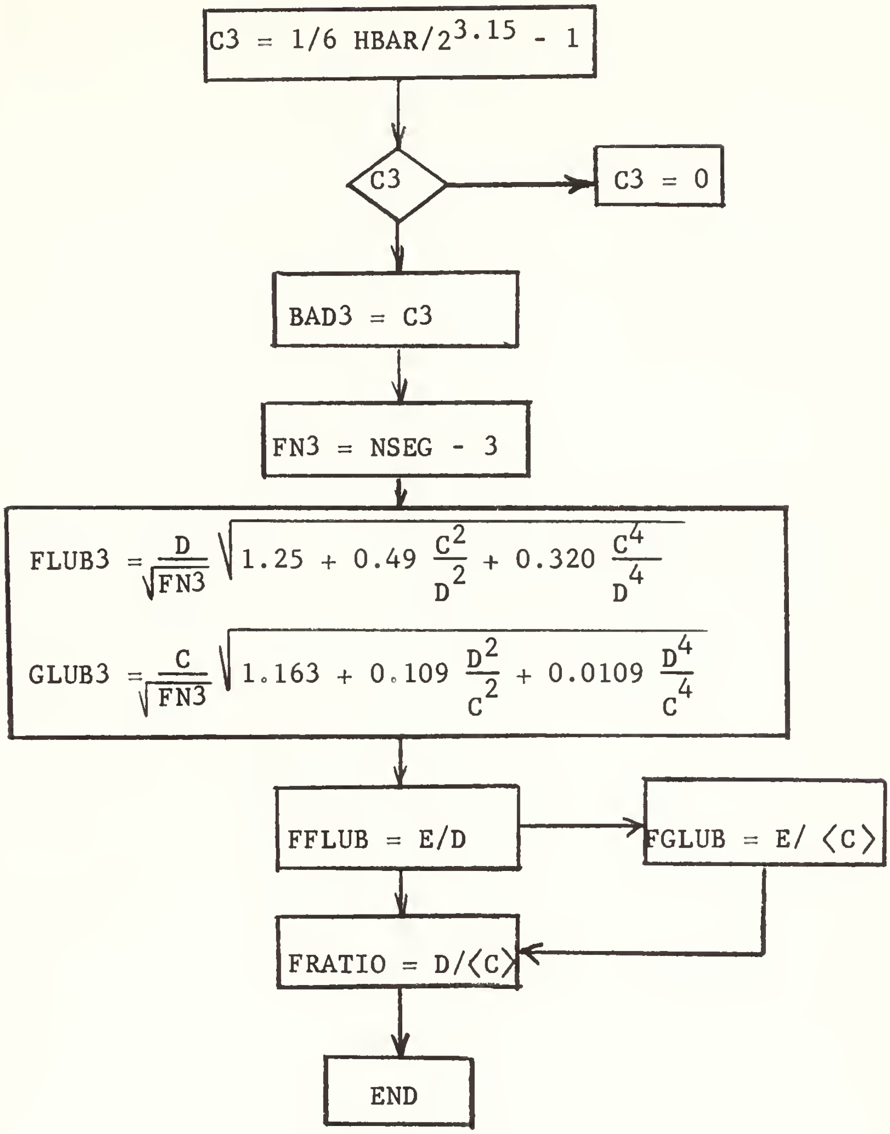
$$FGLUB(M) = E/mC$$

$$FRATIO2(M) = mD/mC$$

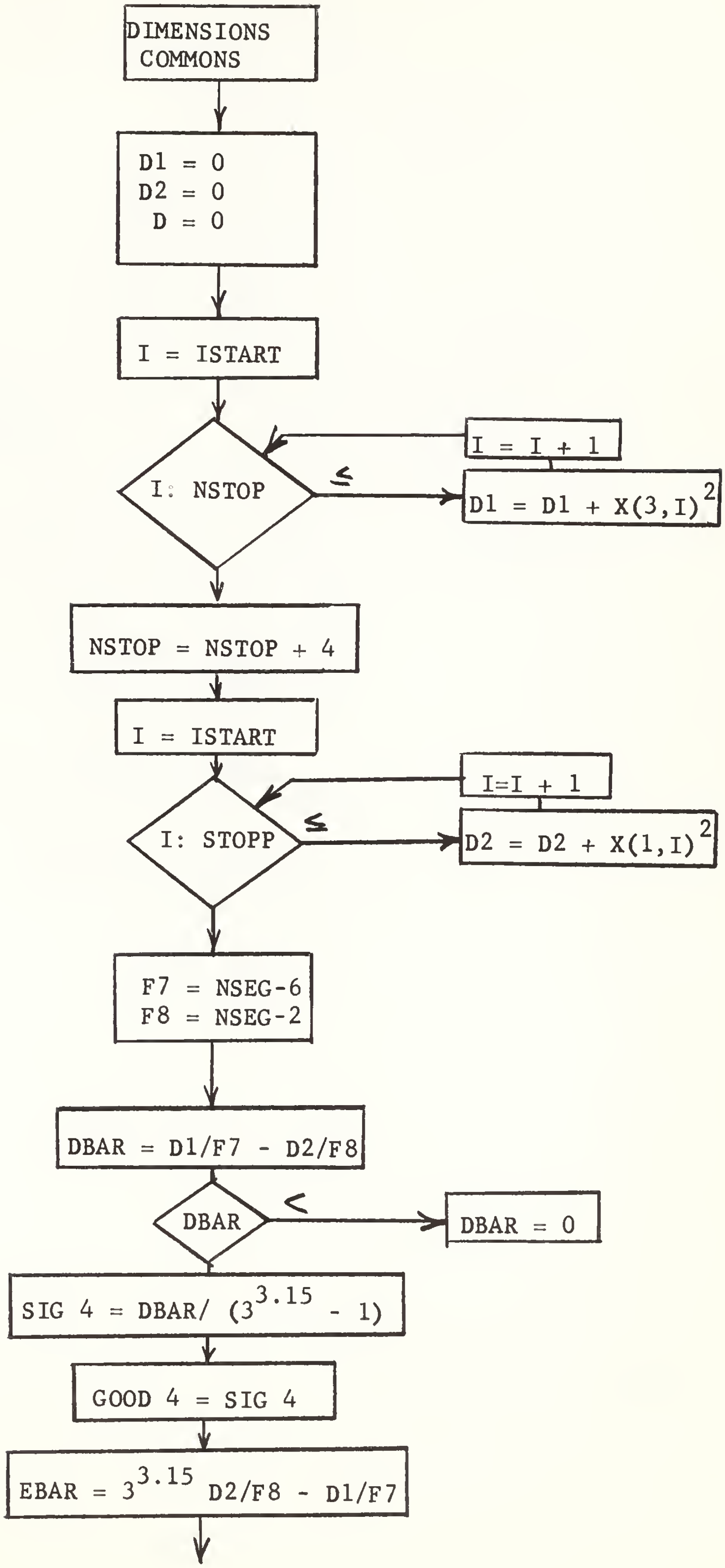
SUBROUTINE MAGIC 3



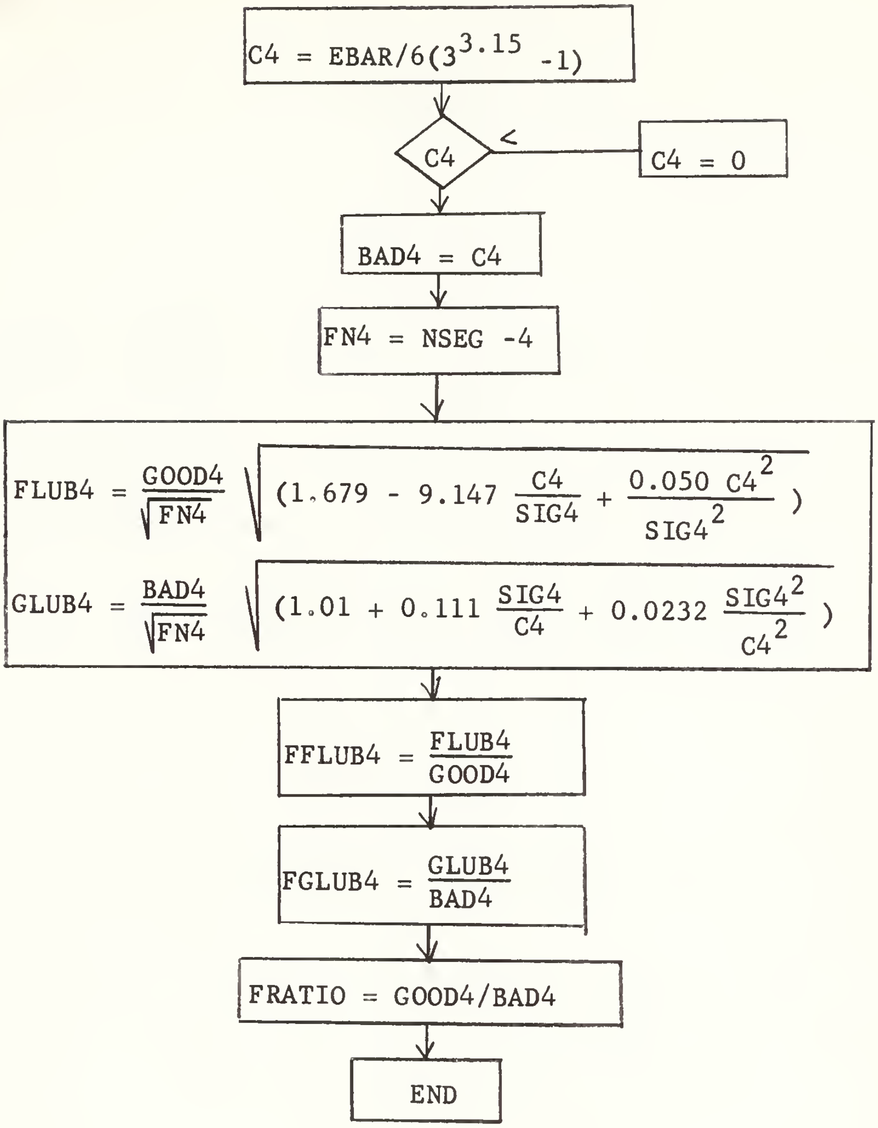
SUBROUTINE MAGIC 3 (con't)



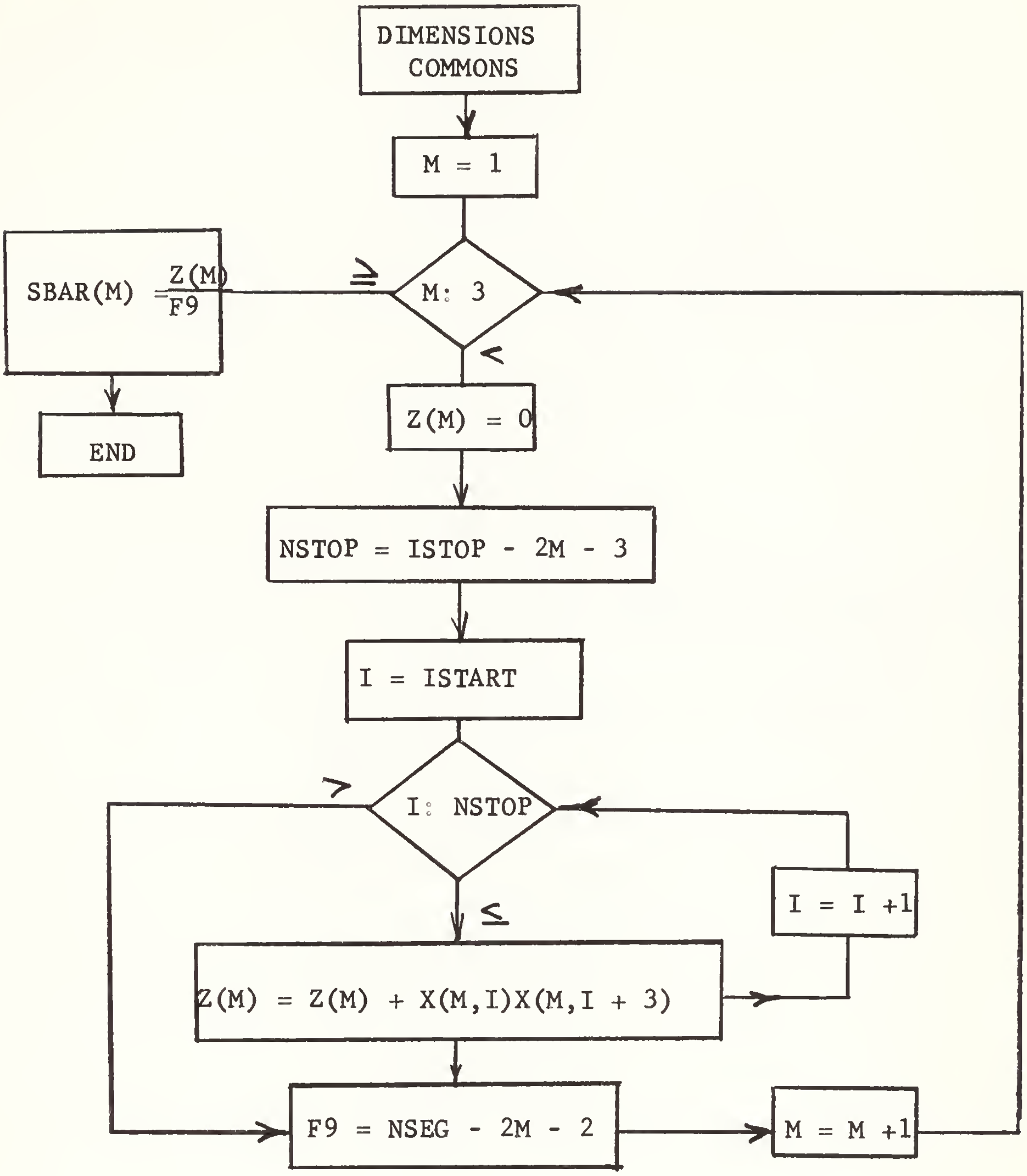
SUBROUTINE MAGIC 4



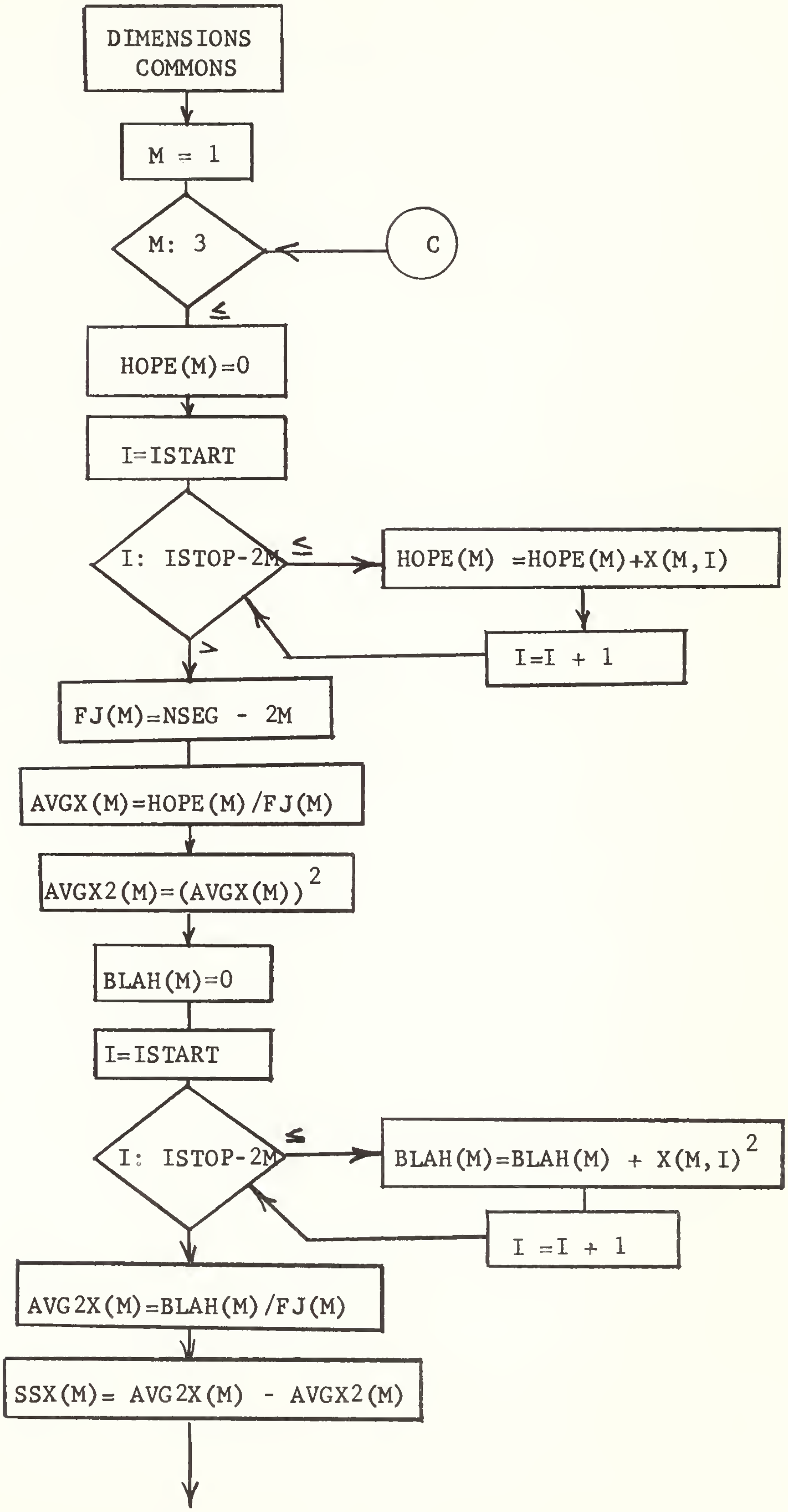
SUBROUTINE MAGIC 4 (con't)



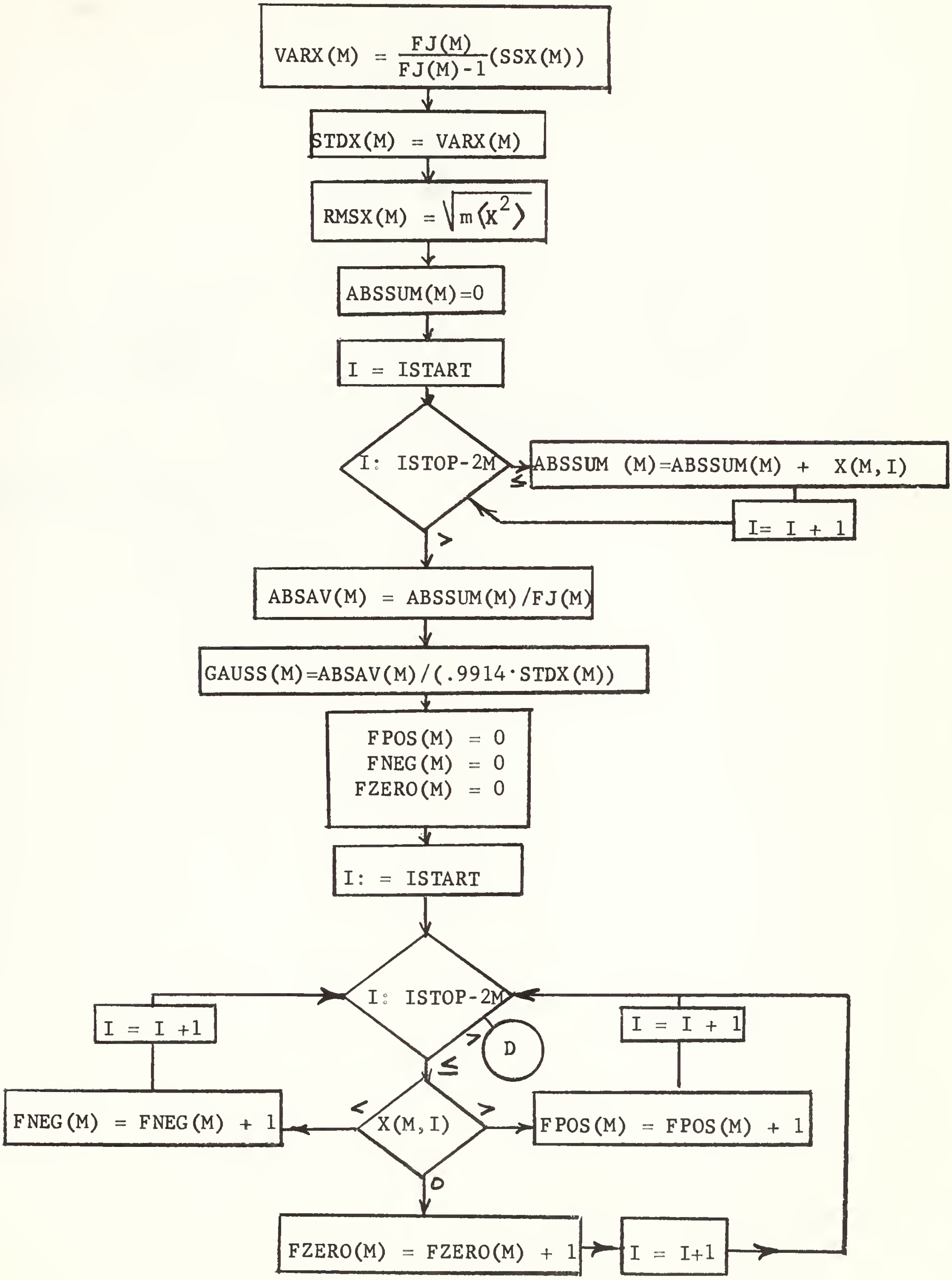
SUBROUTINE MAGIC 5

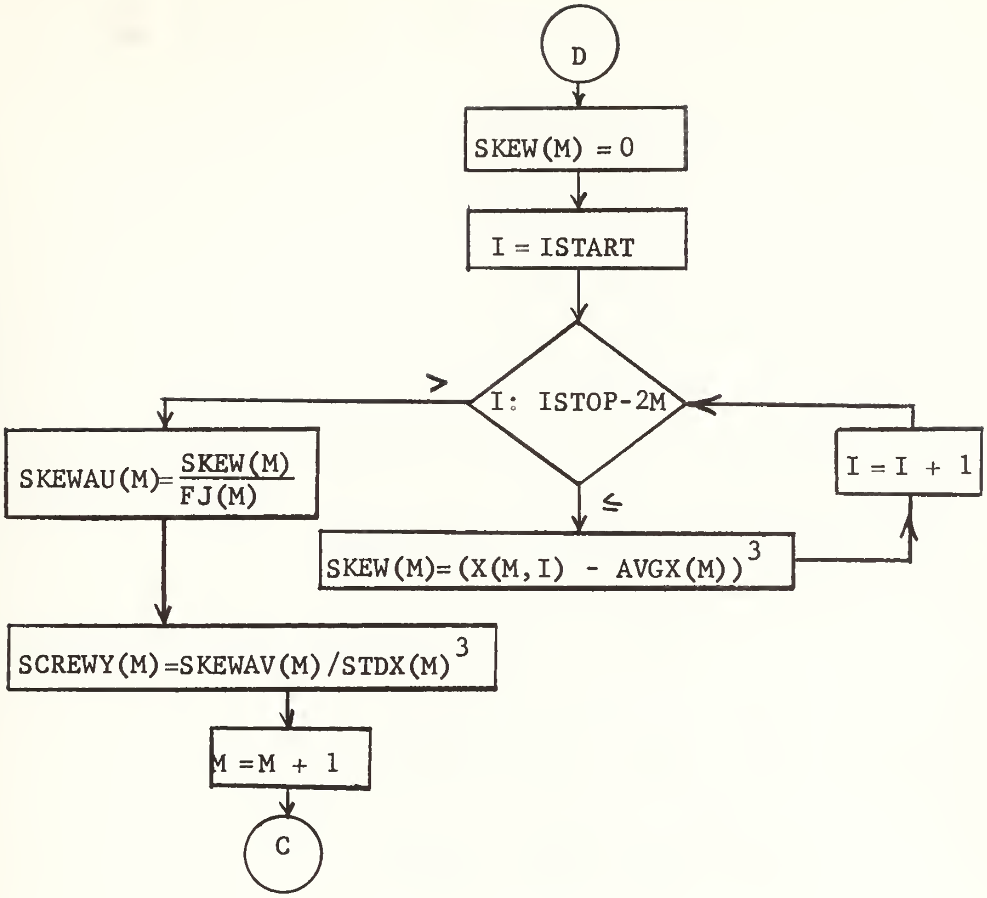


SUBROUTINE HORNY

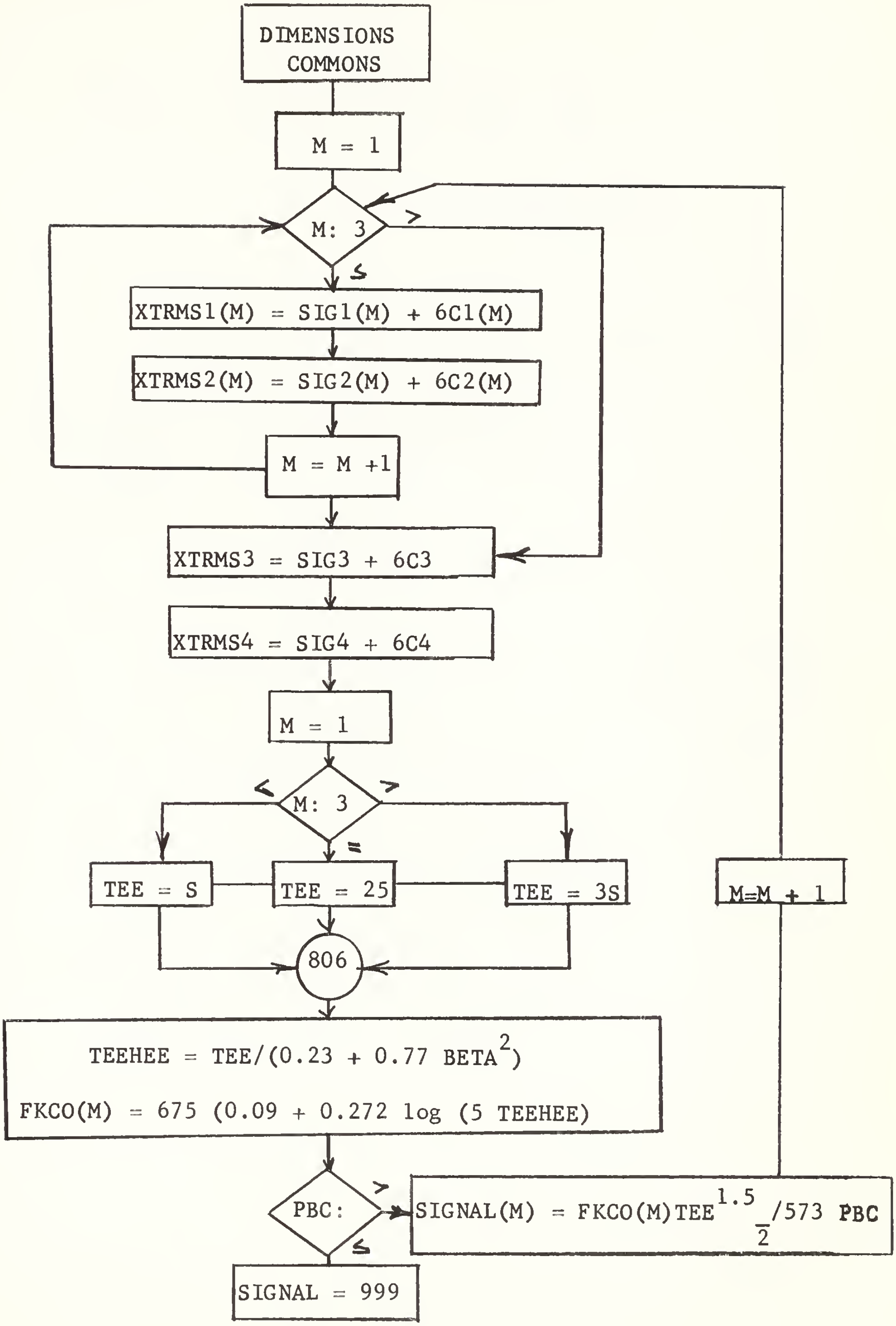


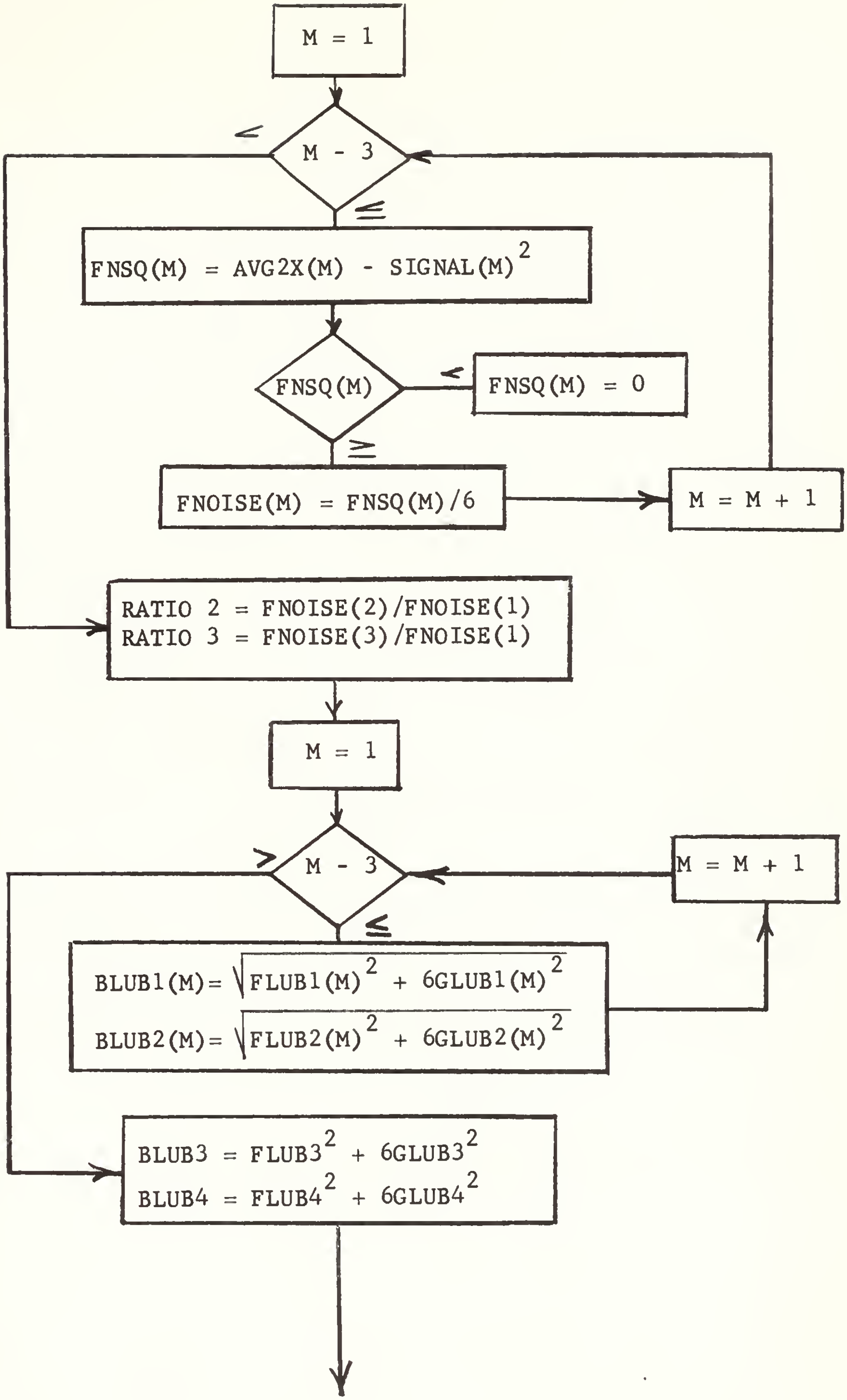
SUBROUTINE HORNY (con't)

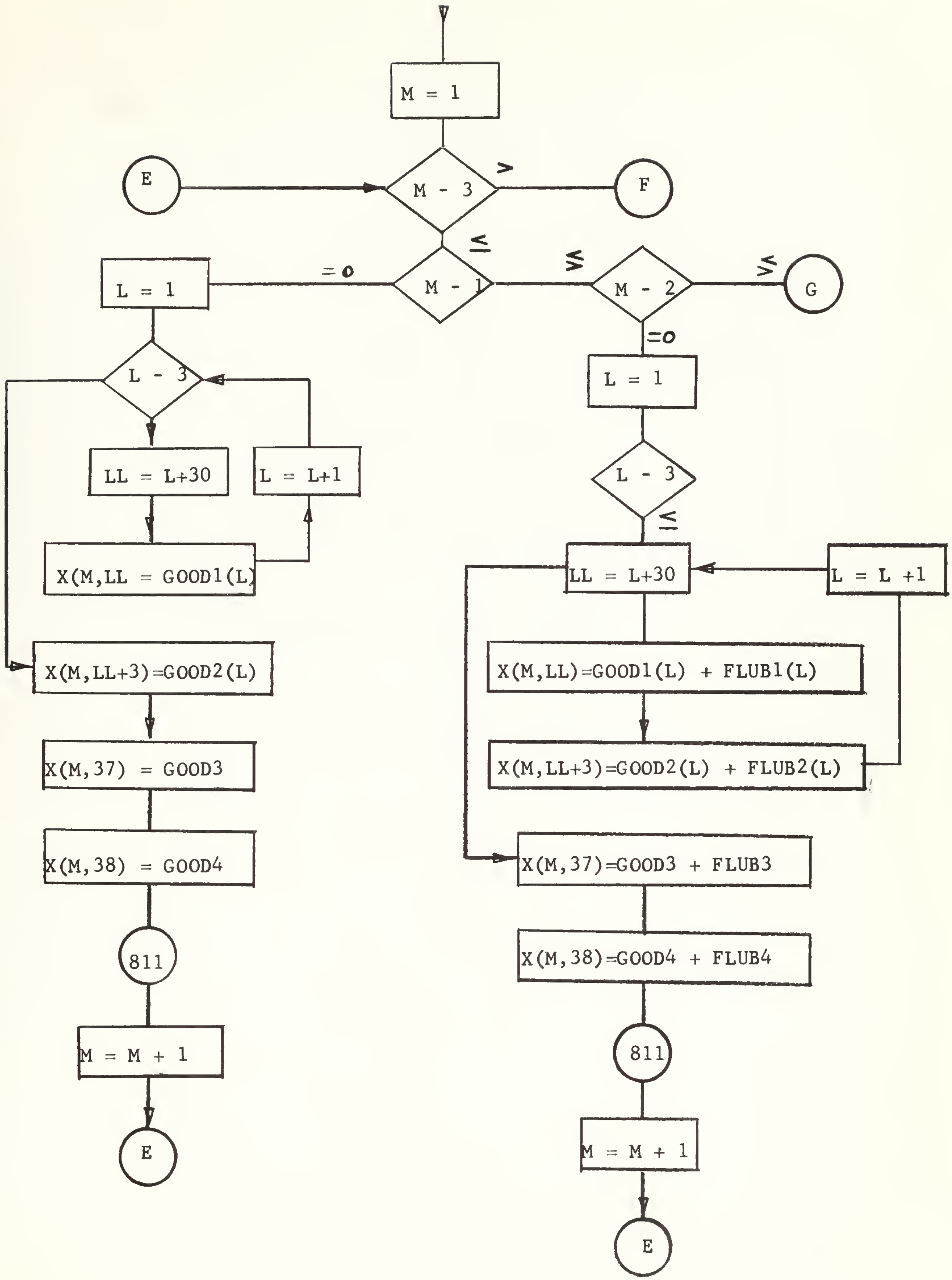


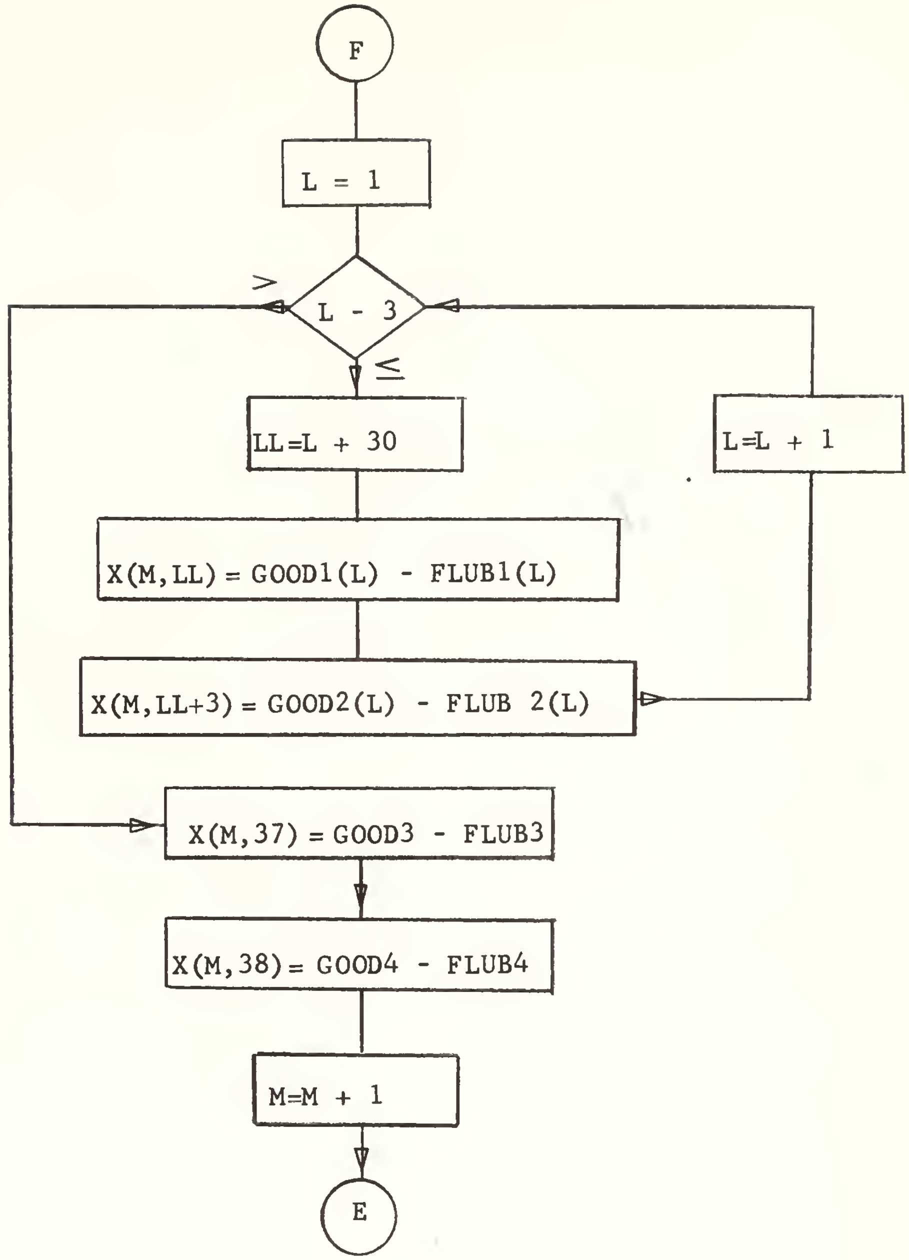

D
SKEW(M) = 0
I = ISTART
I: ISTOP-2M
>
≤
SKEWAU(M)= SKEW(M) / FJ(M)
I = I + 1
SKEW(M)= (X(M,I) - AVGX(M))^3
SCREWY(M)=SKEWAV(M)/STDX(M)^3
M = M + 1
C

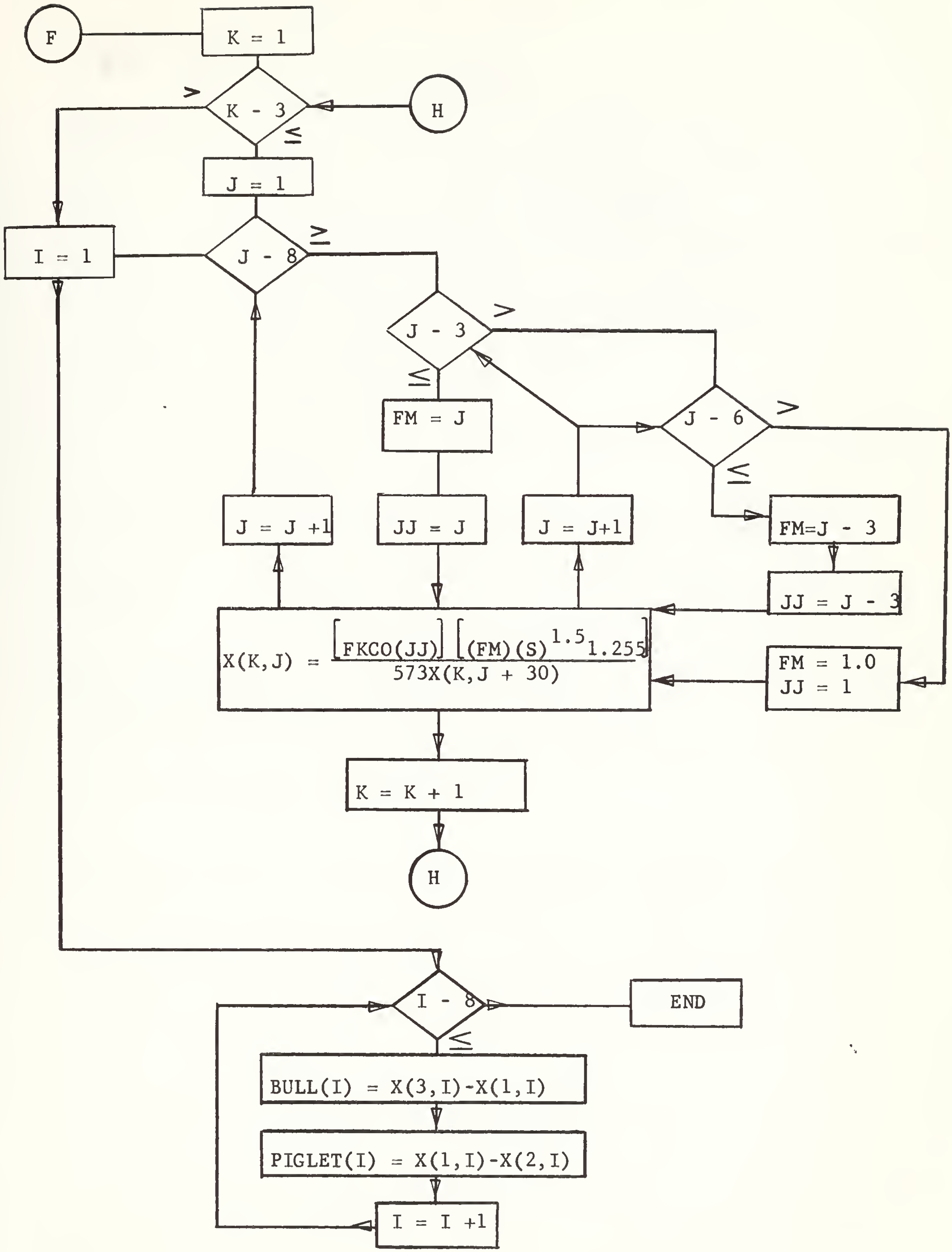
SUBROUTINE COMPARE




M = 1
<
M - 3
≤
FNSQ(M) = AVG2X(M) - SIGNAL(M)2
FNSQ(M)
<
FNSQ(M) = 0
≥
FNOISE(M) = FNSQ(M)/6
M = M + 1
RATIO 2 = FNOISE(2)/FNOISE(1)
RATIO 3 = FNOISE(3)/FNOISE(1)
M = 1
>
M - 3
M = M + 1
≤
BLUB1(M)= √FLUB1(M)2 + 6GLUB1(M)2
BLUB2(M)= √FLUB2(M)2 + 6GLUB2(M)2
BLUB3 = FLUB32 + 6GLUB32
BLUB4 = FLUB42 + 6GLUB42


M = 1
E
M - 3
>
F
≤
L = 1
= 0
M - 1
≥
M - 2
≥
G
= 0
L - 3
LL = L+30
L = L+1
X(M,LL = GOOD1(L)
L = 1
L - 3
≤
LL = L+30
L = L +1
X(M,LL+3)=GOOD2(L)
X(M,LL)=GOOD1(L) + FLUB1(L)
X(M,LL+3)=GOOD2(L) + FLUB2(L)
X(M,37) = GOOD3
X(M,38) = GOOD4
X(M,37)=GOOD3 + FLUB3
X(M,38)=GOOD4 + FLUB4
811
M = M + 1
E
811
M = M + 1
E


F
L = 1
>
L - 3
≤
LL=L + 30
L=L + 1
X(M,LL) = GOOD1(L) - FLUB1(L)
X(M,LL+3) = GOOD2(L) - FLUB 2(L)
X(M,37) = GOOD3 - FLUB3
X(M,38) = GOOD4 - FLUB4
M=M + 1
E


F
K = 1
K - 3
H
>
≤
J = 1
I = 1
J - 8
≥
J - 3
>
≤
FM = J
J - 6
>
≤
J = J +1
JJ = J
J = J+1
FM=J - 3
JJ = J - 3
X(K,J) = [FKCO(JJ)] [(FM)(S)^1.5 1.255] / 573X(K,J + 30)
FM = 1.0
JJ = 1
K = K + 1
H
I - 8
END
≤
BULL(I) = X(3,I)-X(1,I)
PIGLET(I) = X(1,I)-X(2,I)
I = I +1

APPENDIX III

PROGRAM NIRVANA LISTING

```
      PROGRAM NIRVANA
C                         SYMBOL DEFINITIONS
C     Y(I)                I-TH TRACK ORDINATE (INPUT)
C     X(M,I)              I-TH  2ND DIFF, CELL M TIMES UNIT
C     NMAX                NBR OF Y(I) TO BE TAKEN AS A SEGMENT (INPUT)
C     N                   NBR OF Y(I) DATA POINTS IN THE TRACK. LIMIT 200
C     ISTART              INITIAL Y(I) INDEX FOR SEGMENT
C     ISTOP               FINAL Y(I) INDEX FOR SEGMENT
C     KNIX(M)             NBR OF CASTOUTS FOR X(M,I). CASTOUT AT 4 MEAN ABS
C     JOB(I)              CONTROL WORDS FOR SUBROUTINE CALLS
C     GOOD1(M)            RMS SIGNAL BY SUB MAGIC 1, CELL M UNIT
C     BAD1(M)             RMS NOISE BY SUB MAGIC 1,CELL M UNIT
C     FLUB1(M)            STD DEVIATION OF GOOD1(M)
C     GLUB1(M)            STD DEVIATION OF BAD1(M)
C     FFLUB               FRACTIONAL FLUB
C     FGLUB               FRACTIONAL GLUB
C     PBC                 TRACK PBC ,IF KNOWN (INPUT)
C     C1                  SQUARE OF BAD1
C     SIG1                SQUARE OF GOOD
C     SIGNAL              THEORETICAL NOISE FREE  X  CALC FROM PBC
C     NOISE               THAT NEEDED TO GET SIGNAL FROM THE X(M,I)
C     BEST                THE VALUE OF GOOD WITH SMALLEST FFLUB
C     GAUSS               A TEST. EQUALS 1.0 IF X(M,I) ARE GAUSSIAN
C     SCREWY              SKEWNESS COEFF.  3RD MOMENT OVER  STD DEV CUBED
C     FKCO(M)             CALCULATED SCATTERING FACTOR, WITH BETA =1.0
C
C     UNITS OF OUTPUT DATA ARE MICRONS FOR ALL LENGTHS, MEV FOR
C     ENERGIES. INPUT  Y(I) ARE IN TENTHS OF KORISTKA EYEPIECE DIVS
C     CALIBRATION APPEARS AT STATEMENT 102
C     BETA OF TRACK IS ASSUMED TO BE 1.0 TO CALCULATE FKCO
C
C     THIRD DIFFERENCES ARE NOT USED
C
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)      ,FNSQ(3)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
```

```
      READ 1,  (JOB(I), I = 1,10), NMAX
    1 FORMAT  (10I1,7X,I3)
 1000 READ 2 ,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDATE,N,S
    2 FORMAT  (A3,I3,I4,I2,8X,A8,7X,F6.0,A8,2X,I2,A8,I3,2X,F5.0)
      READ 3, (Y(I),I=1,N)
    3 FORMAT (7F10.0)
C
C                    REARRANGE FOR DUPLICATES
C
      ISP2 = NMAX
      IEDIT = 0
    7 I1=0
      NUSED=N
      DELTA=0.
      DO 4 I=1,N
      IF(Y(I)) 6,5,5
    5 Y(I) =Y(I)+DELTA
      Y(I-I1) =Y(I)
      GO TO 4
    6 NUSED =NUSED-1
      I1=I1+1
      DELTA= Y(I)+Y(I-1)
    4 CONTINUE
    8 N = NUSED
      ISTART=0
      ISTOP = 0
   50 NDONE=0
      NNSEG=0
      IF(NMAX -N)52,55,55
   52 NLEFT = N - NDONE
      IF(NMAX - NLEFT)54,53,53
   53 NSEG = NLEFT
      IF (NSEG-15) 60,56,56
      GO TO 56
   54 NSEG = NMAX
      GO TO 56
   55 NSEG = N
   56 NNSEG = NNSEG + 1
      ISTART = ISTOP +1
      ISTOP = ISTOP + NSEG
      BETA =1.00
  100 DO 101 M=1,3
      NSTOP = ISTOP -2*M
      DO 102 I=ISTART,NSTOP
  102 X(M,I) =(Y(I+2*M) -2.*Y(I+M)+Y(I))* 0.0043
      TOTAL =0.
      NIX = 0.
      KNIX(M)=0
  103 DO 104 I=ISTART, NSTOP
  104 TOTAL = TOTAL + ABSF(X(M,I))
      AA= NSEG -2*M -KNIX(M)
      XABAR(M) = TOTAL/AA
      DO 105 L=ISTART,NSTOP
      IF(4.*XABAR(M)-ABSF(X(M,L)))106,105,105
```

```
  106 X(M,L)=0.
      NIX=NIX+1
      KNIX(M)=KNIX(M)+1
  105 CONTINUE
      IF(NIX) 101,101,107
  107 NIX=0
      TOTAL =0.
      GO TO 103

  101 CONTINUE
      GO TO 150
C           STATEMENT  150  STARTS JOB CHECK
C           AT THIS POINT  ALL Y VALUES ARE ARRANGED AND THE
C           SECOND DIFFERENCES X(M,I) ARE  CALCULATED FOR THE SEGMENT
  150 IF(JOB(1)) 151,151,152
  152 CALL MAGIC1
  151 IF(JOB(2)) 154,154,153
  153 CALL MAGIC2
  154 IF(JOB(3)) 156,156,155
  155 CALL MAGIC3
  156 IF(JOB(4)) 158,158,157
  157 CALL MAGIC4
  158 IF(JOB(5)) 160,160,159
  159 CALL MAGIC5
  160 IF (JOB(6)) 162,162,161
  161 CALL MAGIC6
  162 IF (JOB(7)) 164,164,163
  163 CALL HORNY
  164 IF (JOB(8)) 166,166,165
  165 CALL COMPARE
  166 CALL EDIT
   60 NDONE = NDONE + NSEG
      IF (NDONE - N)  52,170,170
  170 IF (NMAX - N) 171, 1001, 1001
 1001 NMAX = ISP2
      GO TO 1000
  171 IF (IEDIT) 1000,172,1000
  172 IEDIT = 1
      NMAX = N
      ISTART = 0
      ISTOP = 0
      GO TO 8
  167 END
      SUBROUTINE MAGIC 1
C           MAGIC1 CALCS USING X(I) AND X(I+1)
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
```

```
    8BULL(8)
     COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
    1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
    2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
    3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
    4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
    5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
    6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
    7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
    8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
    9SMALL,BULL,CLOSE,BEST,ERROR,PV
     COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
     DO 201 M=1,3
     S1(M)=0.
     S2(M)=0.
     NSTOP = ISTOP - 2*M
     DO 202 I = ISTART, NSTOP
 202 S1(M) = S1(M) + X(M,I)**2
     NSTOPP = NSTOP - 1
     DO 203  I = ISTART,NSTOPP
 203 S2(M) = S2(M) +X(M,I)*X(M,I+M)
     F1 = NSEG -2*M
     F2 = F1-1.
     SBAR(M) = S1(M)/F1 +1.5* S2(M)/F2
     SIG1(M) = 8.*SBAR(M)/11.
     IF(SIG1(M)) 204,205,205
 204 SIG1(M)=0.
 205 GOOD1(M)=SQRTF(SIG1(M))
C                 GOOD1 IS RMS  SIGNAL
     GO TO 250
 250 FBAR(M) =S1(M)/F1 -4.*S2(M)/F2
     C1(M) =FBAR(M)/22.
     IF(C1(M)) 251,252,252
 251 C1(M)=0.
 252 BAD1(M) = SQRTF(C1(M))
C                 BAD1  IS RMS NOISE
C
C           START CALC OF ERRORS
C
     FN1 = NSEG -2*M
     IF(FN1)210,211,211
 210 FN1 = 0.0
 211 CONTINUE
     FLUB1(M) =(GOOD1(M)/SQRTF(FN1))*SQRTF(1.02 + 0.926 *  C1(M)/ SIG1(
    1M)+  3.23 * C1(M)**2/ SIG1(M)**2)
     FFLUB1(M) =FLUB1(M)/GOOD1(M)
     GLUB1(M) =(BAD1(M)/SQRTF(FN1))*SQRTF(  1.29 + 0.11  * SIG1(M)/ C1
    1(M) + 0.0074  *SIG1(M) **2/  C1(M)**2)
     FGLUB1(M) = GLUB1(M)/BAD1(M)
     FRATIO1(M)= GOOD1(M)/BAD1(M)
 201 CONTINUE
     END
     SUBROUTINE MAGIC 2
C           MAGIC 2  CALCS  USING  X(I) AND X(I+2)
```

```
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      DO 301 M=1,3
      G1(M)=0.
      G2(M)=0.
      NSTOP = ISTOP - 2*M
      DO 302  I = ISTART,NSTOP
  302 G1(M) = G1(M)+ X(M,I)**2
      NSTOPP = NSTOP - 2
      DO 303  I = ISTART,NSTOPP
  303 G2(M) = G2(M) +X(M,I)*X(M,I+2*M)
      F3 = NSEG -2*M
      F4 = F3 -2.
      GBAR(M) = G2(M)/F4
      FLBAR(M) = G1(M)/F3  -6.*G2(M)/F4
      SIG2(M) = FLBAR(M)
      IF(SIG2(M)) 304,305,305
  304 SIG2(M) =0.
  305 GOOD2(M)= SQRTF(SIG2(M))
C                GOOD2 IS RMS SIGNAL
      GO TO 350
  350 C2(M) =GBAR(M)
      IF (C2(M)) 351,352,352
  351 C2(M)=0.
  352 BAD2(M) =SQRTF(C2(M))
C                BAD2 IS RMS NOISE
C
C          START CALC  OF ERRORS
C
      FN2 = NSEG -2*M
      IF (FN2) 310,311,311
  310 FN2 =0.0
  311 CONTINUE
      FLUB2(M) =(GOOD2(M)/SQRTF(FN2-1.))*SQRTF(10.219 + 76.00* C2(M)/SIG
     12(M) +  506.0 * C2(M)**2/SIG2(M)**2)
```

```
      FFLUB2(M) = FLUB2(M)/GOOD2(M)
      GLUB2(M) = (BAD2(M)/SQRTF(FN2))*SQRTF( 17.75  +    2.0* SIG2(M)/
     1C2(M)+   0.28 * SIG2(M)**2/C2(M)**2)
      FGLUB2(M)= GLUB2(M)/BAD2(M)
      FRATIO2(M)=GOOD2(M)/BAD2(M)
  301 CONTINUE
      END
      SUBROUTINE MAGIC 3
C           MAGIC3 USES  UNIT AND DOUBLE CELLS
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      A1=0.
      A2=0.
      A=0.
      NSTOP = ISTOP - 4
      DO 401  I = ISTART, NSTOP
  401 A1=A1+ X(2,I)**2
      NSTOPP = NSTOP + 2
      DO 402  I = ISTART,NSTOPP
  402 A2 = A2 + X(1,I)**2
      F5 =NSEG -4
      F6 =NSEG -2
      ABAR = A1/F5 -A2/F6
      IF(ABAR)403,404,404
  403 ABAR=0.
  404 SIG3 =        (ABAR/(2.**3.15 -1.))
      GOOD3 =SQRTF(SIG3)
C                GOOD3  IS RMS SIGNAL
      HBAR = (2.**3.15)*A2/F6 - A1/F5
      C3 = HBAR/(6.*(2.**3.15 -1.))
      IF(C3)405,406,406
  405 C3=0.
  406 BAD3=SQRTF(C3)
C                BAD3  IS  RMS  NOISE
C
```

```
C           START  CALC  OF ERRORS
C
      FN3 = NSEG -3
      IF(FN3) 410,411,411
  410 FN3 = 0.0
  411 CONTINUE
      FLUB3 =(GOOD3/SQRTF(FN3))*SQRTF( 1.25 + 0.49  *  C3/SIG3  + 0.320
     1*   C3**2/ SIG3**2)
      FFLUB3 = FLUB3/GOOD3
      GLUB3 =(BAD3/SQRTF(FN3))*SQRTF(1.163 + 0.109  * SIG3/C3   +0.0109
     1*  SIG3**2/   C3**2)
      FGLUB3 = GLUB3/BAD3
      FRATIO3 =GOOD3/BAD3
      END
      SUBROUTINE MAGIC 4
C           MAGIC4  USES UNIT AND  TRIPLE CELLS
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      D1=0.
      D2=0.
      D=0.
      NSTOP = ISTOP - 6
      DO 501  I = ISTART,NSTOP
  501 D1 = D1 + X(3,I)**2
      NSTOPP = NSTOP + 4
      DO 502  I = ISTART, NSTOPP
  502 D2 = D2 + X(1,I)**2
      F7 = NSEG-6
      F8 = NSEG -2
      DBAR = D1/F7 -D2/F8
      IF(DBAR) 503,504,504
  503 DBAR=0.
  504 SIG4 = DBAR/ (3.**3.15 -1.)
      GOOD4 = SQRTF (SIG4)
C                GOOD4 IS RMS SIGNAL
```

```
      EBAR  =(3.**3.15)* D2/F8  -D1/F7
      C4 = EBAR / (6.*(3.**3.15 - 1.))
      IF (C4) 505,506,506
  505 C4=0.
  506 BAD4 = SQRTF(C4)
C                BAD4 IS RMS NOISE
C
C           START  CALC OF ERRORS
C
      FN4 = NSEG -4
      IF (FN4)510,511,511
  510 FN4 =0.0
  511 CONTINUE
      FLUB4  =(GOOD4/SQRTF(FN4))*SQRTF( 1.679 +0.147*  C4/SIG4
     1 + 0.050  *  C4**2/ SIG4**2)
      FFLUB4 =FLUB4/GOOD4
      GLUB4 = (BAD4/SQRTF(FN4))*SQRTF(1.01    + 0.111 * SIG4/C4
     1 + 0.0232  *SIG4**2/C4**2)
      FGLUB4 = GLUB4/ BAD4
      FRATIO4 =GOOD4/BAD4
      END
      SUBROUTINE MAGIC 5
C
C                MAGIC5 USES PRODUCTS X(I) AND X(I+3)
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      DO 601 M=1,3
      Z(M)=0.
      NSTOP = ISTOP - 2*M - 3
      DO 602  I = ISTART, NSTOP
  602 Z(M) =Z(M) +X(M,I)*X(M,I+3)
      F9  = NSEG -2*M -2
  601 ZBAR(M) = Z(M)/F9
      END
      SUBROUTINE MAGIC 6
```

```
      END
      SUBROUTINE HORNY
C          HORNY LOOKS AT THE ACTUAL DISTRIBUTIONS. THIS IS AN
C          IMPORTANT STEP IN REACHING NIRVANA.
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      DO 701  M=1,3
      HOPE(M)=0.
      NSTOP = ISTOP -2*M
      DO 702  I = ISTART,NSTOP
  702 HOPE(M) =HOPE(M) + X(M,I)
      FJ(M) = NSEG -2*M
      IF (FJ(M)) 7002,7002,7003
 7003 AVGX(M) = HOPE(M)/FJ(M)
      AVGX2(M) = AVGX(M)**2
      BLAH(M)=0.
      DO 703  I = ISTART,NSTOP
  703 BLAH(M)= BLAH(M) + X(M,I)**2
      AVG2X(M) = BLAH(M)/FJ(M)
      SSX(M) = AVG2X(M)- AVGX2(M)
      VARX(M)= FJ(M)*SSX(M)/(FJ(M)-1.)
      STDX(M)= SQRTF(VARX(M))
      RMSX(M) = SQRTF(AVG2X(M))
      ABSSUM(M) = 0.0
      FPOS(M) = 0.0
      FNEG(M) = 0.0
      FZERO(M) = 0.0
      DO 704  I = ISTART,NSTOP
      ABSSUM(M) = ABSSUM(M) + ABSF(X(M,I))
      IF (X(M,I))  705,706,707
  705 FNEG(M) = FNEG(M) + 1.0
      GO TO 704
  706 FZERO(M) = FZERO(M) + 1.0
      GO TO 704
  707 FPOS(M) = FPOS(M) + 1.0
```

```
  704 CONTINUE
      ABSAV(M) = ABSSUM(M) / FJ(M)
      GAUSS(M) = ABSAV(M) / (.9914 * RMSX(M))
C                LOOK FOR SKEWNESS NOW
      SKEW(M)=0.
      DO 708  I = ISTART, NSTOP
  708 SKEW(M)= SKEW(M)+ (X(M,I) - AVGX(M))**3
      SKEWAV(M)= SKEW(M)/ FJ(M)
  701 SCREWY(M) = SKEWAV(M)/(STDX(M))**3
C                SCREWY(M) IS THE  DIMENSIONLESS SKEWNESS
C                COEFFICIENT FOR  THE X DISTRIBUTION
 7002 CONTINUE
      END
      SUBROUTINE COMPARE
C          COMPARE  COMPARES THE OBSERVED  X DISTRIBUTION
C          AND THE CALCULATED  VARIANCES AND FINDS THEORETICAL SIGNAL
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)      ,FNSQ(3)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      DO 801  M=1,3
      XTRMS1(M) =SQRTF( SIG1(M) +6.*C1(M))
  801 XTRMS2(M) =SQRTF( SIG2(M) +6.*C2(M))
      XTRMS3    =SQRTF( SIG3 +6.*C3)
      XTRMS4    =SQRTF( SIG4 +6.*C4)
C          THESE SHOULD EQUAL THE STANDARD DEVIATION OF THE
C          OBSERVED  X  DISTRIBUTION  OR  RMS VALUE OF X
C
C          FIND  THE  EXPECTED  RMS SIGNAL FROM KNOWN ENERGY
C          EVALUATE  SCATTERING FACTOR
      DO 802  M= 1,3
      GO TO (803,804,805),M
  803 TEE = S
      GO TO 806
  804 TEE = 2.*S
      GO TO 806
  805 TEE = 3.*S
```

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  806 TEEHEE = TEE/(.23 + .77* BETA**2)
      FKCO(M)= SQRTF(675.*(.09 +.272*LOG10F(5.*TEEHEE)))
      IF(PBC)   807,807,808
  807 SIGNAL =999.
 8007 GO TO 802
  808 SIGNAL(M) =FKCO(M)* TEE**1.5 *SQRTF(3.1416/2.)/(573.*PBC)
  802 CONTINUE
C                SIGNAL(M) IS EXPECTED RMS SIGNAL FOR CELL M TIMES UNIT
C                NOW FIND REQUIRED RMS NOISE
      DO 809 M=1,3
      FNSQ(M) = AVG2X(M) - SIGNAL(M) **2
      IF (FNSQ(M))  850,809,809
  850 FNSQ(M) = 0.0
  809 FNOISE(M) = SQRTF(FNSQ(M)/6.0)
      RATIO2 = FNOISE(2)/FNOISE(1)
      RATIO3 = FNOISE(3)/FNOISE(1)
C          COMPOUND THE CALCULATED VARIANCES OF GOOD AND BAD
      DO 810 M=1,3
      BLUB1(M) =SQRTF(FLUB1(M)**2 + 6.*GLUB1(M)**2)
  810 BLUB2(M) =SQRTF(FLUB2(M)**2 + 6.*GLUB2(M)**2)
      BLUB3 =SQRTF(FLUB3**2 +6.*GLUB3**2)
      BLUB4    =SQRTF(FLUB4**2 +6.*GLUB4**2)
      DO 811 M=1,3
      IF(M-1) 812,813,812
  813 DO 814 L=1,3
      LL=L+30
      X(M,LL) = GOOD1(L)
  814 X(M,LL+3) =GOOD2(L)
      X(M,37)= GOOD3
      X(M,38)= GOOD4
      GO TO 811
  812 IF(M-2) 815,816,815
  816 DO 817 L=1,3
      LL = L+30
      X(M,LL)=GOOD1(L)+FLUB1(L)
  817 X(M,LL+3)=GOOD2(L)+FLUB2(L)
      X(M,37)= GOOD3+FLUB3
      X(M,38)= GOOD4 + FLUB4
      GO TO 811
  815 DO 818 L=1,3
      LL=L+30
      X(M,LL) =GOOD1(L)-FLUB1(L)
  818 X(M,LL+3)= GOOD2(L) -FLUB2(L)
      X(M,37) = GOOD3 - FLUB3
      X(M,38) = GOOD4 - FLUB4
  811 CONTINUE
      DO 820 K=1,3
      DO 821 J=1,8
      IF(J-3) 822, 822, 823
  822 FM=J
      JJ=J
      GO TO 821
  823 IF (J-6) 824,824,825
  824 FM = J-3
```

```
      JJ = J-3
      GO TO 821
  825 FM = 1.0
      JJ=1
  821 X(K,J)=((FKCO(JJ)*(FM*S)**1.5)/(573.0*X(K,J+30)))*1.255
  820 CONTINUE
      DO 830 I=1,8
      BULL (I) = X(3,I) -X(1,I)
  830 PIGLET(I)= X(1,I) - X(2,I)
      END
      SUBROUTINE EDIT
C                EDIT ARRANGES DATA FOR PRINTOUT AND CALCULATES
C                THE BEST VALUE OF PBC FROM MAGIC SUBROUTINES
C
      DIMENSION JOB(10),Y(200),X(3,198),KNIX(3),XABAR(3),S1(3),S2(3),ST(
     13),SBAR(3),SIG1(3),GOOD1(3),FBAR(3),C1(3),BAD1(3),FLUB1(3),FFLUB1(
     23),GLUB1(3),FGLUB1(3),FRATIO1(3),G1(3),G2(3),G(3),GBAR(3),FLBAR(3)
     3,SIG2(3),GOOD2(3),C2(3),BAD2(3),FLUB2(3),FFLUB2(3),GLUB2(3),FGLUB2
     4(3),FRATIO2(3),Z(3),ZBAR(3),HOPE(3),FJ(3),AVGX(3),AVGX2(3),BLAH(3)
     5,AVG2X(3),SSX(3),VARX(3),STDX(3),RMSX(3),ABSSUM(3),ABSAV(3),GAUSS(
     63),FPOS(3),FNEG(3),FZERO(3),SKEW(3),SKEWAV(3),SCREWY(3),XTRMS1(3),
     7XTRMS2(3),FKCO(3),SIGNAL(3),FNOISE(3),BLUB1(3),BLUB2(3),PIGLET(8),
     8BULL(8)
      COMMON  NMAX,ISTACK,IPEL,IEVENT,IPRONG,IPTCL,PBC,ISCNR,ISCOPE,IDAT
     1E,N,S,Y,NUSED,ISTART,ISTOP,NDONE,NNSEG,NLEFT,NSEG,BETA,X,KNIX,SBAR
     2,SIG1,GOOD1,FBAR,C1,BAD1,FLUB1,FFLUB1,GLUB1,FGLUB1,FRATIO1,GBAR,
     3FLBAR,SIG2,GOOD2,C2,BAD2,FLUB2,FFLUB2,GLUB2,FGLUB2,FRATIO2,ABAR,
     4SIG3,GOOD3,HBAR,C3,BAD3,FLUB3,FFLUB3,GLUB3,FGLUB3,FRATIO3,DBAR,
     5SIG4,GOOD4,EBAR,C4,BAD4,FLUB4,FFLUB4,GLUB4,FGLUB4,FRATIO4,ZBAR,
     6AVGX,AVGX2,AVG2X,SSX,VARX,STDX,RMSX,ABSSUM,ABSAV,GAUSS,FPOS,FNEG,
     7FZERO,SKEW,SKEWAV,SCREWY,XTRMS1,XTRMS2,XTRMS3,XTRMS4,TEE,TEEHEE,
     8FKCO,SIGNAL,FNOISE,RATIO2,RATIO3,BLUB1,BLUB2,BLUB3,BLUB4,PIGLET,
     9SMALL,BULL,CLOSE,BEST,ERROR,PV
      COMMON IEDIT, ISP1,ISP2,ISP3,ISP4,ISP5,SP1,SP2,SP3,SP4,SP5,SP6
      IF (NMAX - N)  930,931,931
  930 PRINT 932
  932 FORMAT (55H1  SEGMENT EDIT ONLY   SEE MASTER EDIT FOR TOTAL TRACK
     1//)
      GO TO 9000
  931 IF (IEDIT)  933,934,933
  934 PRINT 935
  935 FORMAT (42H1 COMPLETE EDIT, ONLY ONE SEGMENT IN TRACK//)
      GO TO 9000
  933 PRINT 936
  936 FORMAT (34H1  MASTER EDIT OF SEVERAL SEGMENTS//)
      PRINT 9000
 9000 FORMAT (27H  UNIT OF LENGTH IS MICRONS/ 71H  ESTIMATES ARE GIVEN A
     1S VALUE, STD DEV, FRACTIONAL SD, SIG/NOISE RATIO//)
      PRINT 900, IEVENT,ISCNR, NNSEG
  900 FORMAT (10H  EVENT = ,I4,36X,10HSCANNER = ,A8,27X,14HSEGMENT NBR =
     1 I1)
      PRINT 901,ISTACK,IDATE,NSEG
  901 FORMAT (10H  STACK = ,A3,40X,7HDATE = ,A8,27X,14HSEG LENGTH  = I4)
      PRINT 902,IPEL,S
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902 FORMAT (10H  PLATE = ,I3,82X,14HUNIT CELL    = F5.0)
    PRINT 903,IPRONG,IPTCL,N
903 FORMAT (10H  PRONG = ,I2,29X,19HTYPE OF PARTICLE = ,A8,27X,14HTOTA
   1L DATA  = ,I3//)
    PRINT 904, PBC
904 FORMAT (24H  INPUT VALUE FOR PBC = ,F6.0,4H MEV//)
    PRINT 905
905 FORMAT (46HDATA CALCULATED FROM THE OBSERVED DISTRIBUTION/)
    PRINT 906
906 FORMAT(34X,3HM=1,17X,3HM=2,17X,3HM=3)
    PRINT 907,AVGX(1),AVGX(2),AVGX(3),RMSX(1),RMSX(2),RMSX(3),STDX(1),
   1STDX(2),STDX(3),ABSAV(1),ABSAV(2),ABSAV(3),GAUSS(1),GAUSS(2),
   2GAUSS(3),FPOS(1),FPOS(2),FPOS(3),FNEG(1),FNEG(2),FNEG(3),FZERO(1),
   3FZERO(2),FZERO(3),KNIX(1),KNIX(2),KNIX(3),SCREWY(1),SCREWY(2),
   4SCREWY(3)
907 FORMAT(10X,6HMEAN X,12X,F10.2,10X,F10.2,10X,F10.2/10X,5HRMS X,13X,
   1F10.2,2(10X,F10.2)/10X,13HSTD DEVIATION,5X,F10.2,2(10X,F10.2)/10X,
   210HMEAN ABS X,8X,F10.2,2(10X,F10.2)/10X,10HGAUSS TEST,8X,F10.2,
   32(10X,F10.2)/10X,10HNBR POS  X,8X,F10.0,2(10X,F10.0)/10X,10HNBR NE
   4G  X,8X,F10.0,2(10X,F10.0)/10X,10HNBR ZERO X,8X,F10.0,2(10X,F10.0)
   5/10X,11HNBR CASTOUT,7X,I10,2(10X,I10)/10X,8HSKEWNESS,10X,F10.2,2(1
   60X,F10.2)///)
    PRINT 908
908 FORMAT (30H  COMPARISON BETWEEN ESTIMATES// )
    PRINT 909
909 FORMAT(15X,5HINPUT,10X,7HMAGIC 1,17X,7HMAGIC 2,17X,7HMAGIC 3,
   117X,7HMAGIC 4/)
    PRINT 910,SIGNAL(1),GOOD1(1),FLUB1(1),FFLUB1(1),FRATIO1(1),
   1GOOD2(1),FLUB2(1),FFLUB2(1),FRATIO2(1),GOOD3,FLUB3,FFLUB3,
   2FRATIO3,GOOD4,FLUB4,FFLUB4,FRATIO4
910 FORMAT (12H  RMS SIGNAL,3X,F5.2,5X,4(F4.2,X,F4.2,1X,F4.2,X,F4.1,
   15X))
    PRINT 911, (SIGNAL(M),GOOD1(M),FLUB1(M),FFLUB1(M),FRATIO1(M),
   1GOOD2(M),FLUB2(M),FFLUB2(M),FRATIO2(M),M=2,3)
911 FORMAT (15X,F5.2,5X, 2(F4.2,X,F4.2, X,F4.2,X,F4.1,5X))
    PRINT 912, FNOISE(1),BAD1(1),GLUB1(1),FGLUB1(1),FRATIO1(1),BAD2(1)
   1,GLUB2(1),FGLUB2(1),FRATIO2(1),BAD3,GLUB3,FGLUB3,FRATIO3,BAD4,
   2GLUB4,FGLUB4,FRATIO4
912 FORMAT (12H  RMS  NOISE,3X,F5.2,5X, 4(F4.2,X,F4.2,1X,F4.2,X,
   1F4.1,5X))
    PRINT 911, (FNOISE(M),BAD1(M),GLUB1(M),FGLUB1(M),FRATIO1(M),
   1BAD2(M),GLUB2(M),FGLUB2(M),FRATIO2(M),  M=2,3)
    PRINT 913, RMSX(1),XTRMS1(1),XTRMS2(1),XTRMS3,XTRMS4
913 FORMAT (7H  RMS X,8X,F5.2,10X,F5.2,3(20X,F5.2))
    PRINT 914, RMSX(2),XTRMS1(2),XTRMS2(2)
914 FORMAT (15X,F5.2,10X,F5.2,20X,F5.2)
    PRINT 915, RMSX(3),XTRMS1(3),XTRMS2(3)
915 FORMAT (15X,F5.2,10X,F5.2,20X,F5.2 ///)
916 PRINT 917
917 FORMAT(44HCALCULATED VALUES OF PBC FROM MAGIC ROUTINES//)
    PRINT 918 ,(X(1,L),BULL(L),PIGLET(L),X(3,L),X(2,L),L=1,8)
918 FORMAT(7H  PV = ,F6.0,4H MEV, 6H PLUS ,F6.0,7H MINUS ,F6.0,
   110H  RANGE    ,F6.0,4H TO ,F6.0)
    PRINT 940,ZBAR(1),ZBAR(2),ZBAR(3)
```

```
940 FORMAT (14HOMAGIC 5 SAYS ,3(F6.3,2X)/)
    PRINT 941
941 FORMAT(55HTHIS WORK CHEERFULLY PERFORMED BY NIRVAVA YELLOW        )
    END
    END
```

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